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Sensitivity Analysis of Costs in Grey Transportation Problems

F. Pourofoghi*, D. Darvishi Salokolaei

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Abstract The parameters of the transportation problem in the real world can be presented mostly as grey numbers. Therefore, after solving the grey transportation problem and reaching the optimal answer, sensitivity analysis of cost coefficients of a grey transportation problem is discussed. In this paper, using the definitions of center and width of interval grey numbers, a new method for the sensitivity analysis of cost coefficients of a grey transportation problem is presented. In this way, it can be determined the ranges of costs in the grey transportation problem such that its optimal basis is invariant. The proposed method is also explained with an example.

Keyword: Sensitivity; Grey System; Transportation Problem.

1 Introduction

The transportation issue is one of the important optimization issues that has many applications, including supply chain improvement. Basically, the goal of the Transportation Problem (TP) is to minimize the total cost of transporting goods from origins to destinations. If we consider the parameters of the TP accurately, we will have a normal TP that can be solved by different methods and the solution of the problem can be found. But in real everyday problems, due to the changes in the conditions governing the world, it is very difficult to determine the exact parameters of the TP [1]. Therefore, we need to make decisions under conditions of uncertainty. Different approaches such as interval, fuzzy, rough and random numbers have been developed for decision making in imprecise conditions [2]. Annie Christi and Kalpana [3] investigated multi-objective fuzzy transportation problems. Yeola and Jahav [4] have proposed a method to solve the multi-objective TP in which a fuzzy programming technique with a fuzzy linear membership function with different costs is used. Many researchers have researched in the field of TP in imprecise environments, to study their results, refer to [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. But when the number of data is small and the information is incomplete, the use of the above approaches becomes less useful. Therefore, to make decisions in such situations, Deng [17] introduced the concept of Grey Systems (GS) theory. By presenting this theory, the Grey Linear Programming problem (GLP) was introduced. Various methods have been proposed to solve GLPP [18, 19, 20, 21, 22]. The Grey Transportation Problem (GTP) is a generalization of the GLPP in imprecise

* **Corresponding Author.** (✉)
E-mail: f_pourofoghi@pnu.ac.ir (F. Pourofoghi)

F. Pourofoghi
Department of Mathematics, Payame Noor University, Tehran, Iran.

D. Darvishi Salokolaei
Department of Mathematics, Payame Noor University, Tehran, Iran.

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environments. The GTP does not require distribution information for its parameters, because Grey Numbers (GN) are suitable for imprecise inputs. Pourofoghi et al. [23] proposed a way to solve GTP. Nasseri et al. [24] proposed a direct approach to solve the grey assignment problem based on grey calculations. Examining the changes made after solving the model and reaching the optimal answer is called Sensitivity Analysis (SA). SA is one of the most important fields of optimization. Many researchers have studied the SA of the common TP [25, 26, 27, 28, 5]. Darvishi et al. [29] presented the SA of GLPP. Kavitha [30] has studied the SA of the interval TP. Therefore, in this paper, using the concepts of center and width of GN, we present a method for SA of GTP objective function coefficients. In this method, we first analyze the sensitivity of the Central Transportation Problem (CTP) and the Width Transportation Problem (WTP), and finally, by using the relationships between the center and the width of interval GN, we analyze the sensitivity of the GTP. An example is provided to illustrate the idea of the center-width method.

This paper is presented in 6 sections as follows. After the introduction, in Section 2, the basic knowledge about GST that is needed in the following sections is stated. The formulation of GTP model is presented in Section 3. The proposed algorithm for SA of the GTP is presented in Section 4. In Section 5, we present and solve a numerical example to express the efficiency and understanding of the proposed method. Discussion is provided in Section 6. Finally, Section 7 contains conclusions and recommendations.

2 Preliminaries Grey System Theory

In this section, we provide the necessary definitions for the study of GST to investigate the SA of GTP. GST is a method to deal with problems in imprecise environments. GST studies imprecise systems with little and unknown information. GN is one of the main concepts of GST that play a significant role in the study of uncertainty. There are different types of GN, in this paper we use interval GN.

Definition 2.1 [31] Each GN is represented by a symbol $\otimes x \in [\underline{x}, \overline{x}]$, where in $\underline{x}$ lower bound and $\overline{x}$ an upper bound.

Definition 2.2 [31] Center and Width of Grey Number

1. Center $(\otimes_c x)$ of GN $\otimes x \in [\underline{x}, \overline{x}]$ is defined as follows:

$$\otimes_c x = \frac{\underline{x} + \overline{x}}{2}. \tag{1}$$

2. Width $(\otimes_w x)$ of GN $\otimes x \in [\underline{x}, \overline{x}]$ is defined as follows:

$$\otimes_w x = \frac{\overline{x} - \underline{x}}{2}. \tag{2}$$

Remark 2.1 [31] For the GN $\otimes x \in [\underline{x}, \overline{x}]$, we have:

$$\underline{x} = \otimes_c x - \otimes_w x , \quad \overline{x} = \otimes_c x + \otimes_w x . \tag{3}$$

Definition 2.3 [31] Let $\otimes x_1 \in \left[\underline{x}_1, \overline{x}_1\right]$ and $\otimes x_2 \in \left[\underline{x}_2, \overline{x}_2\right]$ be two interval GNs.

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$$\begin{aligned}
&\otimes x_1 + \otimes x_2 = \left[\underline{x}_1 + \underline{x}_2, \bar{x}_1 + \bar{x}_2\right] \\
&\otimes x_1 - \otimes x_2 = \otimes x_1 + \left(-\otimes x_2\right) = \left[\underline{x}_1 - \bar{x}_2, \bar{x}_1 - \underline{x}_2\right] \\
&\otimes x_1 \times \otimes x_2 = \left[\min\left\{\underline{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2\right\}, \max\left\{\underline{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2\right\}\right] \\
&k.\otimes x \in \begin{cases} \left[k\underline{x}, k\bar{x}\right] & k > 0 \\ \left[k\bar{x}, k\underline{x}\right] & k < 0 \end{cases}
\end{aligned} \tag{4}$$

3 Grey Transportation Problem

In real-world transportation problems, it is better to use TP with unknown parameters to build a model. Inaccuracy in the parameters means that the information of these parameters is incomplete. However, even with incomplete information, the model used is usually able to determine the gray values for the parameters. Therefore, using GTP is more suitable for modeling and solving real-world problems [32]. A GTP is created when the TP table contains gray numbers. The GTP is shown in Table 1.

Table 1 Matrix of the GTP

Sources \ Destinations	1	2	…	j	…	n	Supply
1	$\otimes c_{11}$	$\otimes c_{12}$	…	$\otimes c_{1j}$	…	$\otimes c_{1n}$	$\otimes s_1$
2	$\otimes c_{21}$	$\otimes c_{22}$	…	$\otimes c_{2j}$	…	$\otimes c_{2n}$	$\otimes s_2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$\otimes c_{i1}$	$\otimes c_{i2}$	…	$\otimes c_{ij}$	…	$\otimes c_{in}$	$\otimes s_i$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$\otimes c_{n1}$	$\otimes c_{n2}$	…	$\otimes c_{nj}$	…	$\otimes c_{nn}$	$\otimes s_m$
Demand	$\otimes d_1$	$\otimes d_2$	…	$\otimes d_j$	…	$\otimes d_n$	

The LP model for the GTP becomes as follows:

$$\begin{aligned}
&Min \ \otimes Z =_G \sum_{i=1}^{m}\sum_{j=1}^{n} \otimes c_{ij} \otimes x_{ij} \\
&s.t. \qquad \sum_{j=1}^{n} \otimes x_{ij} =_G \otimes s_i\,, \quad i = 1,\ 2, \ldots., m \\
&\qquad\qquad \sum_{i=1}^{m} \otimes x_{ij} =_G \otimes d_j\,, \quad j = 1,\ 2, \ldots., n \\
&\qquad\qquad \otimes x_{ij} \geq_G \otimes 0, \ \ i = 1,\ 2, \ldots., m;\ j = 1,\ 2, \ldots., n.
\end{aligned} \tag{5}$$

Where;

Grey unit cost: $\otimes c_{ij}$, i=1,2,...,m , j=1,2,...,n.

The amount of grey supply: $\otimes s_i$, i=1,2,...,m.

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The amount of grey demand: $\otimes d_j$, j=1,2,...,n.
Also; $\otimes c_{ij} >_G \otimes 0$, $\otimes d_j >_G \otimes 0$, $\otimes s_i >_G \otimes 0$.

Therefore:

$$
\begin{aligned}
& Min\ \otimes Z =_G \sum_{i=1}^{m}\sum_{j=1}^{n} \otimes\left[\underline{c}_{ij},\overline{c}_{ij}\right]\times\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right] \\
& s.t. \qquad \sum_{j=1}^{n}\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right] =_G \otimes\left[\underline{s}_i,\overline{s}_i\right], \quad i=1,\ 2,....,\ m \\
& \qquad\quad \sum_{i=1}^{m}\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right] =_G \otimes\left[\underline{d}_j,\overline{d}_j\right], \quad j=1,\ 2,....,\ n \\
& \qquad\quad \underline{x}_{ij} \geq 0\ \ ,\ \overline{x}_{ij} \geq 0,\ \ i=1,\ 2,....,\ m\ ;\ \ j=1,\ 2,....,\ n
\end{aligned} \tag{6}
$$

So that;
$\underline{c}_{ij}, \overline{c}_{ij}, \underline{s}_i, \overline{s}_i, \underline{d}_j, \overline{d}_j > 0.$

Definition 3.1 The $\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right]$, *for all* $i=1,2,...,m$ *and* $j=1,2,...,n$, is said to be a feasible solution of GTP if they satisfy the equations (6).

Definition 3.2 A feasible solution $\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right]$, *for all* $i=1,2,...,m$ *and* $j=1,2,...,n$, is said to be an optimal solution of GTP if

$$
\sum_{i=1}^{m}\sum_{j=1}^{n}\otimes\left[\underline{c}_{ij},\overline{c}_{ij}\right]\times\otimes\left[\underline{x}_{ij},\overline{x}_{ij}\right] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}\otimes\left[\underline{c}_{ij},\overline{c}_{ij}\right]\times\otimes\left[\underline{y}_{ij},\overline{y}_{ij}\right] \tag{7}
$$

for i=1,2,...,m and j=1,2,...,n and for all feasible solution $\otimes\left[\underline{y}_{ij},\overline{y}_{ij}\right]$, *for all* $i=1,2,...,m$ *and* $j=1,2,...,n$.

Definition 3.3 A grey feasible solution to a GTP with *m* sources and *n* destinations is said to be a grey basic feasible solution if and only if

1. The sum of the allocated values in each row is equal to the supply quantity,
2. The sum of the allocated values in each column is equal to the amount of demand,
3. The sum of supply and demand is equal to the sum allocated $\left(\sum_{i=1}^{m}\otimes s_i = \sum_{j=1}^{n}\otimes d_j\right)$,
4. The number of basic variables is equal to $(m+n-1)$.

This theorem is given to express the relationship between the optimal solution to the GTP and the solutions to the two problems obtained from it.

Theorem 3.1 [23] Let the set $\left\{\otimes_c x^0_{ij}\ ,\ for\ all\ \ i=1,2,...,m\ \ and\ \ j=1,2,...,n\right\}$ is an optimal solution of CTP of GTP:

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$$\begin{aligned} & Min\ \otimes_c Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \otimes_c c_{ij} \times \otimes_c x_{ij} \\ & s.t. \quad \sum_{j=1}^{n} \otimes_c x_{ij} = \otimes_c s_i\ , \quad i = 1,\ 2,....,\ m \\ & \qquad \sum_{i=1}^{m} \otimes_c x_{ij} = \otimes_c d_j\ , \quad j = 1,\ 2,....,\ n \\ & \qquad \otimes_c x_{ij} \geq 0\ \ ,\ \ i = 1,\ 2,....,\ m\ ;\ j = 1,\ 2,....,\ n \end{aligned} \tag{8}$$

and the set $\left\{ \otimes_w x^0{}_{ij}\ ,\ for\ all\ \ i = 1,2,...,m\ and\ \ j = 1,2,...,n \right\}$ is an optimal solution of the WTP of GTP:

$$\begin{aligned} & Min\ \otimes_w Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \otimes_w c_{ij} \times \otimes_w x_{ij} \\ & s.t. \quad \sum_{j=1}^{n} \otimes_w x_{ij} = \otimes_w s_i\ , \quad i = 1,\ 2,....,\ m \\ & \qquad \sum_{i=1}^{m} \otimes_w x_{ij} = \otimes_w d_j\ , \quad j = 1,\ 2,....,\ n \\ & \qquad \otimes_w x_{ij} \geq 0\ \ ,\ \ i = 1,\ 2,....,\ m\ ;\ j = 1,\ 2,....,\ n \end{aligned} \tag{9}$$

Then, the set $\left\{ \left[\otimes_c x^0{}_{ij} - \otimes_w x^0{}_{ij}, \otimes_c x^0{}_{ij} + \otimes_w x^0{}_{ij} \right], for\ all\ \ i = 1,2,...,m\ and\ \ j = 1,2,...,n \right\}$ is an optimal solution of the GTP.

4 Sensitivity Analysis

The study of changes made after solving the model and reaching the optimal answer, is called SA.

In this section, we discuss the following two aspects of SA for the transportation problem:

1. Changing the objective function coefficient of a non-basic variable.
2. Changing the objective function coefficient of a basic variable

4.1 Sensitivity Analysis of Costs in Grey Transportation Problem

For analyzing the sensitivity of (i,j)-th cost of the GTP, we need to the SA of (i,j)-th cost of the CTP and the WTP of GTP.

Now, to analyze the sensitivity range of (i,j)-th cost of the CTP, replace $\otimes_c c_{ij}$ by $\otimes_c c_{ij} + \Delta_c$. Calculate the minimum and maximum values of Δ_c and denote by $\Delta_c{}^0$ and $\Delta_c{}^1$, respectively so that the current solution remains optimal provided. $\Delta_c^0 c_{ij} \leq \otimes_c c_{ij} \leq \Delta_c^1 c_{ij}$

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Now, to analyze the sensitivity range of (i,j)-th cost of the WTP, replace $\otimes_w c_{ij}$ by $\otimes_w c_{ij} + \Delta_w$. Calculate the minimum and maximum values of Δ_w and denote by $\Delta_w{}^0$ and $\Delta_w{}^1$, respectively so that the current solution remains optimal provided. $\Delta_w^0 c_{ij} \le \otimes_w c_{ij} \le \Delta_w^1 c_{ij}$

Then, using the results obtained from the SA of the central and width problem and the remark 2.1, the minimum and maximum grey interval values of the (i,j)-th cost $\otimes\left[\underline{c}_{ij}, \overline{c}_{ij}\right]$ is $\otimes\left[\Delta_c^0 c_{ij} - \Delta_w^0 c_{ij}, \Delta_c^0 c_{ij} + \Delta_w^0 c_{ij}\right]$ and $\otimes\left[\Delta_c^1 c_{ij} - \Delta_w^1 c_{ij}, \Delta_c^1 c_{ij} + \Delta_w^1 c_{ij}\right]$, respectively.

4.2 Central- Width Algorithm

The steps of the central-width algorithm are as follows:

Step1: A study of SA of the CTP.

i. Using the least cost cell (or inspection) method and the MODI method, the optimal answer for CTP it is obtained as follows.
ii. Study the SA of the CTP. For a non-basic cells.
iii. Study the SA of the CTP. For a basic cells.
iv. Obtain the minimum and maximum values of $\otimes_c c_{ij}$, say $\Delta_c^0 c_{ij}$ and $\Delta_c^1 c_{ij}$ respectively so that the optimal basis to the problem is not changed and $\Delta_c^0 c_{ij} \le \otimes_c c_{ij} \le \Delta_c^1 c_{ij}$.

Step2: A study of SA of the WTP.

i. Using the least cost cell (or inspection) method and the MODI method, the optimal answer for WTP it is obtained as follows.
ii. Study the SA of the WTP. For a non-basic cells.
iii. Study the SA of the WTP. For a basic cells.
iv. Obtain the minimum and maximum values of $\otimes_w c_{ij}$, say $\Delta_w^0 c_{ij}$ and $\Delta_w^1 c_{ij}$ respectively so that the optimal basis to the problem is not changed and $\Delta_w^0 c_{ij} \le \otimes_w c_{ij} \le \Delta_w^1 c_{ij}$.

Step3: A study of SA of the GTP.

The minimum and maximum grey interval values of the (i,j)th cost $\otimes\left[\underline{c}_{ij}, \overline{c}_{ij}\right]$ is $\otimes\left[\Delta_c^0 c_{ij} - \Delta_w^0 c_{ij}, \Delta_c^0 c_{ij} + \Delta_w^0 c_{ij}\right]$ and $\otimes\left[\Delta_c^1 c_{ij} - \Delta_w^1 c_{ij}, \Delta_c^1 c_{ij} + \Delta_w^1 c_{ij}\right]$ respectively.

In the following, an example is provided for further explanation.

5 Numerical Example

In this section, the performance of the proposed algorithm is shown by providing an example.

Example 5.1 Consider the following GTP.

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Table 2 Matrix of the GTP

Sources \ Destinations	$\otimes d_1$	$\otimes d_2$	$\otimes d_3$	Supply
$\otimes s_1$	$\otimes[4,8]$	$\otimes[2,8]$	$\otimes[2,6]$	$\otimes[10,90]$
$\otimes s_2$	$\otimes[2,4]$	$\otimes[3,9]$	$\otimes[2,6]$	$\otimes[5,55]$
Demand	$\otimes[5,45]$	$\otimes[5,35]$	$\otimes[5,65]$	$\otimes[15,145]$

Solution: Let us formulate the example, as follows:

$$Min \ \otimes Z =_G \otimes[4,8]\times\otimes x_{11} + \otimes[2,8]\times\otimes x_{12} + \otimes[2,6]\times\otimes x_{13} + \otimes[2,4]\times\otimes x_{21} + \otimes[3,9]\times\otimes x_{22} + \otimes[2,6]\times\otimes x_{23}$$

$$s.t. \quad \sum_{j=1}^{3} \otimes x_{1j} =_G \otimes[10,90]$$

$$\sum_{j=1}^{3} \otimes x_{2j} =_G \otimes[5,55]$$

$$\sum_{i=1}^{2} \otimes x_{i1} =_G \otimes[5,45]$$

$$\sum_{i=1}^{2} \otimes x_{i2} =_G \otimes[5,35]$$

$$\sum_{i=1}^{2} \otimes x_{i3} =_G \otimes[5,65]$$

$$\otimes x_{ij} \geq_G \otimes 0, \quad i=1,\ 2;\ j=1,\ 2,3.$$

Step1: A study of SA of the CTP.

Now, the CTP of the given GTP is given below:

Table 3 Matrix of the CTP

Sources \ Destinations	$\otimes_c d_1$	$\otimes_c d_2$	$\otimes_c d_3$	Supply	
$\otimes_c s_1$	6	5	4	50	$u_1 = 0$
$\otimes_c s_2$	3	6	4	30	$u_2 = 0$
Demand	25	20	35	80	
	$v_1 = 3$	$v_2 = 5$	$v_3 = 4$		

Using the least cost cell (or inspection) method and the MODI method, the optimal answer for CTP it is obtained as follows.

$\otimes_c x_{12} = 20$, $\otimes_c x_{13} = 30$,$\otimes_c x_{21} = 25$,$\otimes_c x_{23} = 5.$

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i. Study the SA of the CTP. For a non-basic cells.

Consider the SA of the CTP in the non-basic cell ($\otimes_c s_1, \otimes_c d_1$).

Table 4 Matrix of the CTP

Sources \ Destinations	$\otimes_c d_1$	$\otimes_c d_2$	$\otimes_c d_3$	Supply	
$\otimes_c s_1$	$6+\Delta$	5	4	50	$u_1 = 0$
$\otimes_c s_2$	3	6	4	30	$u_2 = 0$
Demand	25	20	35	80	
	$v_1 = 3$	$v_2 = 5$	$v_3 = 4$		

According to the relationship $\otimes_c \bar{c}_{ij} = \otimes_c c_{ij} - u_i - v_j \quad , \; i = 1,2, \, j = 1,2,3$, it is clear that Chang the objective function coefficient of a non-basic variable only affect the optimal condition. Therefore, in order to keep the answer obtained in the final panel, let us do the following:

$$\otimes_c \bar{c}_{ij} = \otimes_c c_{ij} - u_i - v_j \geq 0 \quad , \; i = 1,2, \, j = 1,2,3$$

$$x_{11} : \otimes_c \bar{c}_{11} = \otimes_c c_{11} - u_1 - v_1 \geq 0 \Rightarrow 6 + \Delta - 0 - 3 \geq 0 \Rightarrow \Delta \geq -3 \Rightarrow \Delta \in [-3, +\infty)$$

Therefore, $\otimes_c \bar{c}_{11}$ varies from 3 to $+\infty$.

ii. Study the SA of the CTP. For a basic cells.

Consider the SA of the CTP in the basic cell ($\otimes_c s_1, \otimes_c d_2$).

Table 5 Matrix of the CTP

Sources \ Destinations	$\otimes_c d_1$	$\otimes_c d_2$	$\otimes_c d_3$	Supply	
$\otimes_c s_1$	6	$5+\Delta_1$	4	50	$u_1 = 0$
$\otimes_c s_2$	3	6	4	30	$u_2 = 0$
Demand	25	20	35	80	
	$v_1 = 3$	$v_2 = 5 + \Delta_1$	$v_3 = 4$		

First we calculate the new values u_i and v_j, $\; i = 1,2, \, j = 1,2,3$.

And use these values to price out all non-basic variables.

Given the relationship $\otimes_c \bar{c}_{ij} = \otimes_c c_{ij} - u_i - v_j \quad , \; i = 1,2, \, j = 1,2,3$, we have,

$$\otimes_c \bar{c}_{ij} = \otimes_c c_{ij} - u_i - v_j \geq 0 \quad , \; i = 1,2, \, j = 1,2,3$$

$$x_{11} : \otimes_c \bar{c}_{11} = \otimes_c c_{11} - u_1 - v_1 = 6 - 0 - 3 = 3 \geq 0$$

$$x_{22} : \otimes_c \bar{c}_{22} = \otimes_c c_{22} - u_2 - v_2 \geq 0 \Rightarrow 6 - 0 - 5 - \Delta \geq 0 \Rightarrow \Delta \leq 1 \Rightarrow \Delta \in (-\infty, 1]$$

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Therefore, $\otimes_c \bar{c}_{12}$ varies from 0 to 6.

In the same way, we can find the sensitivity range of all costs in the CTP problems.

Table 6 Matrix of the sensitivity range of all costs in the CTP problems

cell	Minimum limit	Original value	Maximum limit
(1,1)	3	6	$+\infty$
(1,2)	0	5	6
(1,3)	3	4	7
(2,1)	0	3	6
(2,2)	5	6	$+\infty$
(2,3)	1	4	5

Step2: A study the SA of the WTP.

Now, the WTP of the given GTP is given below:

Table 7 Matrix of the WTP

Sources \ Destinations	$\otimes_w d_1$	$\otimes_w d_2$	$\otimes_w d_3$	Supply	
$\otimes_w s_1$	2	3	2	40	$u_1 = 0$
$\otimes_w s_2$	1	3	2	25	$u_2 = 0$
Demand	20	15	30	65	
	$v_1 = 1$	$v_2 = 3$	$v_3 = 2$		

Using the least cost cell (or inspection) method and the MODI method, the optimal answer for WTP it is obtained as follows:

$\otimes_w x_{12} = 15$, $\otimes_w x_{13} = 25$, $\otimes_w x_{21} = 20$, $\otimes_w x_{23} = 5$.

i. Study the SA of the WTP. For a non-basic cells.

Consider the SA of the WTP in the non-basic cell ($\otimes_w s_1, \otimes_w d_1$).

Table 8 Matrix of the WTP

Sources \ Destinations	$\otimes_w d_1$	$\otimes_w d_2$	$\otimes_w d_3$	Supply	
$\otimes_w s_1$	$2+\Delta_2$	3	2	40	$u_1 = 0$
$\otimes_w s_2$	1	3	2	25	$u_2 = 0$
Demand	20	15	30	65	
	$v_1 = 1$	$v_2 = 3$	$v_3 = 2$		

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According to the relationship $\otimes_w \bar{c}_{ij} = \otimes_w c_{ij} - u_i - v_j$, $i = 1,2, j = 1,2,3$, it is clear that Chang the objective function coefficient of a non-basic variable only affect the optimal condition. Therefore, in order to keep the answer obtained in the final panel, let us do the following:

$$\otimes_w \bar{c}_{ij} = \otimes_c c_{ij} - u_i - v_j \geq 0 \quad , \; i = 1,2, j = 1,2,3$$

$$x_{11} : \otimes_w \bar{c}_{11} = \otimes_w c_{11} - u_1 - v_1 \geq 0 \Rightarrow 6 + \Delta_2 - 0 - 1 \geq 0 \Rightarrow \Delta_2 \geq -5 \Rightarrow \Delta_2 \in [-5, +\infty)$$

Therefore, $\otimes_w \bar{c}_{11}$ varies from 1 to $+\infty$.

ii. Study the SA of the WTP. For a basic cells.

Consider the SA of the WTP in the basic cell ($\otimes_w s_1, \otimes_w d_2$).

Table 9 Matrix of the WTP

Sources \ Destinations	$\otimes_w d_1$	$\otimes_w d_2$	$\otimes_w d_3$	Supply	
$\otimes_w s_1$	2	$3+\Delta$	2	40	$u_1 = 0$
$\otimes_w s_2$	1	3	2	25	$u_2 = 0$
Demand	20	15	30	65	
	$v_1 = 1$	$v_2 = 3+\Delta$	$v_3 = 2$		

First we calculate the new values u_i and v_j $i = 1,2, j = 1,2,3$ and use these values to price out all non-basic variables.

Given the relationship $\otimes_w \bar{c}_{ij} = \otimes_w c_{ij} - u_i - v_j$, $i = 1,2, j = 1,2,3$, we have,

$$\otimes_w \bar{c}_{ij} = \otimes_w c_{ij} - u_i - v_j \geq 0 \quad , \; i = 1,2, j = 1,2,3$$

$$x_{11} : \otimes_w \bar{c}_{11} = \otimes_w c_{11} - u_1 - v_1 \geq 0 \Rightarrow 2 - 0 - 0 \geq 0 \Rightarrow 2 \geq 0$$

$$x_{22} : \otimes_w \bar{c}_{22} = \otimes_w c_{22} - u_1 - v_1 \geq 0 \Rightarrow 3 - 0 - 3 - \Delta \geq 0 \Rightarrow \Delta \leq 0 \Rightarrow \Delta \in (-\infty, 0]$$

Therefore, $\otimes_w \bar{c}_{12}$ varies from 0 to 3.

In the same way, we can find the sensitivity range of all costs in the WTP problems.

Table 10 Matrix of the sensitivity range of all costs in the WTP problems

cell	Minimum limit	Original value	Maximum limit
(1,1)	2	2	$+\infty$
(1,2)	0	3	3
(1,3)	2	2	3
(2,1)	0	1	2
(2,2)	3	3	$+\infty$
(2,3)	2	2	4

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Step 3: A study the SA of the GTP.

Using Table 6 and Table 10 and remark 2.1, we will have the minimum and maximum costs of the GTP, provided that the optimal solution is invariant in the Table 11.

Table 11 Matrix of the sensitivity range of all costs in the GTP problems

cell	Minimum limit	Original value	Maximum limit
(1,1)	$[2,4]$	$[4,8]$	$(0,+\infty)$
(1,2)	$[0,0]$	$[2,8]$	$[3,9]$
(1,3)	$[1,5]$	$[2,6]$	$[4,10]$
(2,1)	$[0,2]$	$[2,4]$	$[6,6]$
(2,2)	$[2,8]$	$[3,9]$	$(0,+\infty)$
(2,3)	$[0,4]$	$[2,6]$	$[1,9]$

6 Discussion

This Table shows that, if the cost of transporting goods from origin 1 to destination 1 is within the range $[2,4]$ of $(0,+\infty)$, the optimal basis of the grey transportation problem will remain constant.

If the cost of transporting goods from origin 1 to destination 3 is within the range $[0,0]$ of $[3,9]$, the optimal basis of the grey transportation problem will remain constant.

If the cost of transporting goods from origin 1 to destination 3 is within the range $[1,5]$ of $[4,10]$, the optimal basis of the grey transportation problem will remain constant.

If the cost of transporting goods from origin 2 to destination 1 is within the range $[0,2]$ of $[6,6]$, the optimal basis of the grey transportation problem will remain constant.

If the cost of transporting goods from origin 2 to destination 2 is within the range $[2,8]$ of $(0,+\infty)$, the optimal basis of the grey transportation problem will remain constant.

If the cost of transporting goods from origin 2 to destination 3 is within the range $[0,4]$ of $[1,9]$, the optimal basis of the grey transportation problem will remain constant.

7 Conclusion

The TP is widely used in real life problems. Due to the non-determinism of the parameters of the TP in real life problems, there are different methods to solve them in non-deterministic conditions. One of these methods is the use of GTP. In this paper, using the concepts of center and width of GN, we were presented a method for SA of GTP objective function coefficients. In this method, we first analyzed the sensitivity of the central transportation problem and the width transportation problem, and finally, by using the relationships between the center and the width of interval GN, we analyzed the sensitivity of the GTP. In this way, it is possible to find ranges for the grey cost coefficients of the GTP, where any change in those ranges does

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not change the optimal value of the objective function. Considering the price fluctuations in the market, finding these ranges helps managers to make the right decisions.

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