

# A New Method for Ranking Distribution Companies with Several Scenarios Data by Using DEA/MADM

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**Abstract** In Data Envelopment Analysis, uncertain data are the inseparable part of real models. Natural models usually deal with uncertain and probable data. Many researchers prioritize these kinds of data. For instance, they study fuzzy data, interval data, probabilistic models etc. In this article, we proposed a method in which the decision making units are uncertain in their inputs and outputs. In the proposed method, it is supposed that the inputs and outputs have different scenarios with specific probability occurrence. In this article, applying the VIKOR technique rather than point estimate, the decision making units whose inputs and outputs have different scenarios with specific probability occurrence are ranked. It finds the compromise ranking list and the compromise solution obtained with the given weights from a set of alternatives in the presence of conflicting criteria. In this article, we combine the DEA and the VIKOR method to rank DMUs with different scenarios with a specified probability for input and output data. To illustrate the ability of proposed combined method, a numerical example of 38 Iranian electricity distribution companies is considered.

**Keywords:** Power Distribution, Data Envelopment Analysis, VIKOR Method, Scenario Base Data.

## 1 Introduction

Data envelopment analysis (DEA) is a mathematical programming method for evaluating performance and measuring peer decision making units (DMUs). This method discovers the optimal combination of inputs and outputs for independent and peer decision making units (DMUs). Consider  $n$  DMUs with nonnegative vectors of  $x_j = (x_{1j}, x_{2j}, \dots, x_{sj})$  as an input vector and  $y_j = (y_{1j}, y_{2j}, \dots, y_{rj})$ ,  $j=1, \dots, n$ , as an output vector, In which  $x_{ij}$  shows the value of  $i$ th input and  $y_{ij}$  represents the value of  $i$ th output for DMU <sub>$j$</sub> ,  $j=1, \dots, n$ . Efficiency score of DMU <sub>$d$</sub>  is defined as  $e_d(u, v) = y_d u^t / x_d v^t$ . This ratio is the weighted sum of its outputs to

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the weighted sum of its inputs. In addition,  $u$  and  $v$  are the row vectors of input and output weights. Podinovski and Athanassopoulos [1] and Podinovski [2] suggested  $f_d(u, v)$  as below:

$$f_d(u, v) = e_d(u, v) / \max\{e_j(u, v) \mid u, v \geq 0 \mid j = 1, \dots, n\} \quad (1)$$

which is called relative efficiency of  $DMU_d$ . It is obvious that  $f_d^*$  does not exceed 1 for any  $DMU_j$ . The optimal solution for the model (1) originates input and output vectors which generally differ from one DMU to another. This model is the same as the CCR DEA model, introduced by Charnes, Cooper, and Rhodes [3]. A number of different DEA models, after proposing the CCR model, have been introduced by Seiford and Thrall [4], Seiford [5], and Cooper, Seiford and Tone [6]. These models have wide applications in various performance evaluation problems. In all traditional methods, all inputs and outputs are in values which are exact numerically values form while observed values of inputs and outputs data in real problems are usually uncertain, interval or based on different scenarios. In order to study this issue, different approaches have been proposed in the literature. O'Neal, Ozcan, & Yanking [7] offered the exclusion of the DUMs, to have vague data in order to calculate efficiency. This affects efficiency score and relative efficiency of other DMUs. Therefore, it is not acceptable to factoring out DMUs from our calculation [8]. In some approaches, imputation technique is used (e.g. data average for other DMUs). Using this method in DEA may cause misleading results because of stability problems, where a unit accepting a small change may alter its classification status from efficient to inefficient or vice versa. Accordingly, it is not a reliable method for DEA [6].

A Stochastic approach is also applied for uncertain data in DEA. Stochastic programming has undergone many theoretical developments since the 1950s, starting with the pioneering works of Dantzig [9] and Beale [10]. Nevertheless, this approach suffers from its drawbacks. One such drawback with this method is determining probability distribution function in the absence of sufficient empirical evidence [11]. Another approach, which was proposed by Kuosmanen [12], applies dummy variables instead of missing data. The output is considered to be zero and input data is considered to be a relatively large number with regard to the other input numbers and using a weight limit to reduce the impact of that data on the efficiency of other DMUs. Two other approaches to vague or indistinct data are using the fuzzy and interval DEA, developed by Sengupta [11] and Cooper, Park and Yu [13] respectively.

Cooper et al. [13] have developed an interval method by converting the DEA model to the form of linear programming allowing the combination of exact and non-exact data. Assessing upper and lower limits of the relative efficiency of DMUs is one of the difficulties in the interval method. Regardless to this difficulty. Despite this difficulty, some researchers have developed numerous interval methods [14-19].

What motivated Sengupta [11] to present a fuzzy method and a fuzzy linear programming as a practical approach in such circumstances were the uncertainties over the stability of efficiency frontier and probabilistic feasibility of inequality constraints in DEA. In a fuzzy approach, several mathematical programming methods including probabilistic planning and  $\alpha$ -cut approaches are applied to evaluate the relative efficiency of DMUs. Although there is a sharp growth in the complexity of fuzzy method in some circumstances, many researchers studied on fuzzy DEA [20- 30].

By considering uncertainty in output parameters for the performance assessment of electricity distribution companies, Sadjadi and Omrani [31] have proposed a robust model for DEA considering uncertainty in output parameters for the performance assessment of electricity distribution companies. Shokouhi, Hatami-Marbini, Tavana, and Saati [32] have proposed an approach based on a robust optimization model where the input and output

parameters are constrained to be within an uncertainty set under the assumption of a worst case efficiency defined by the uncertainty set and its supporting constraint. Hafezalkotob et al. [33] proposed RDEA which is based upon the discrete robust optimization approaches proposed by Mulvey et al. [34] that utilizes probable scenarios to capture the effect of ambiguous data in the case study.

Multi-criteria optimization is the method of identifying the best solution which is feasible for alternatives with some criteria representing different effects. Practical problems are often considered by several non-commensurable and different criteria and there may be no solution which can simultaneously satisfy all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker's preferences. The compromise solution was established by Yu [35] and Zeleny [36] for a problem with conflicting criteria and it can help the making units to achieve a final solution. The compromise solution is a feasible solution, it means it is the closest possibility to the ideal solution, and compromise in this term means an agreement which has been established by mutual concessions.

A multi attribute decision making (MADM) problem can be defined as below:

**Table 1** A multi attribute decision making (MADM) problem

|       | $C_1$                      | $C_2$    | ... | $C_n$    |
|-------|----------------------------|----------|-----|----------|
| $A_1$ | $f_{11}$                   | $f_{12}$ | ... | $f_{1n}$ |
| $A_2$ | $f_{21}$                   | $f_{22}$ | ... | $f_{2n}$ |
| ...   | ...                        | ...      | ... | ...      |
| $A_m$ | $f_{m1}$                   | $f_{m2}$ |     | $f_{mn}$ |
|       | $W=[w_1, w_2, \dots, w_n]$ |          |     |          |

while decision making units must choose a possible alternative from  $A_1, A_2, \dots, A_m$ , alternative performance is evaluated by criteria  $C_1, C_2, \dots, C_n$ .  $f_{ij}$  is the rating of alternative  $A_i$  with respect to criterion  $C_j$ ,  $w_j$  is the weight of criterion  $C_j$  [37-39]. In classical MCDM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague; consequently, and decision maker (DM) cannot predict their preference with exact numerical values. Conditions like these, specifying the exact value of the attributes is difficult or impossible. Therefore, in order describe and treat imprecise and uncertain elements which are existing in a decision making problem, fuzzy and stochastic approaches are applied frequently.

## 2 The CCR Model and Efficiency

The model CCR is a model in input nature for  $n$  DMUs. Each DMU is specified by some inputs  $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$  and outputs  $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$  in which all the inputs and outputs are nonnegative and all the DMUs are independent and peer. Efficiency score of  $DMU_d$  is achieved by the model 2 which is known as CCR model:

Min  $\theta_d$

$$\begin{aligned} \sum_{j=1}^n \lambda_j x_{ij} &\leq \theta x_{io} \quad i=1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} &\geq y_{ro} \quad r=1, \dots, s \\ \lambda_j &\geq 0 \quad j=1, \dots, n \end{aligned} \quad (2)$$

$\theta=1$ ,  $\lambda_j = 0$  ( $j \neq d$ ) and  $\lambda_d = 1$  is feasible solution for the model 2. Therefore, the optimal solution for this model which is shown by  $\theta^*$  is smaller or equal to 1. It is clear that if  $\theta_d^*$  is equal to 1, DMU is efficient and if it is smaller than 1, it is inefficient. The Dual for the model 2 is as follows:

$$\begin{aligned} \text{Max } E_{dd} &= \sum_{r=1}^s u_{rd} y_{rd} \\ \sum_{i=1}^m v_{id} x_{id} &= 1 \\ \sum_{r=1}^s u_{rj} y_{rj} - \sum_{i=1}^m v_{ij} x_{ij} &\leq 0 \quad j=1, \dots, n \\ u_{rd} &\geq 0, \quad v_{id} \geq 0 \quad r=1, \dots, s \quad i=1, \dots, m \end{aligned} \quad (3)$$

In this model,  $E_{dd}^*$  is equal to  $\theta_d^*$  in the model CCR for DMU<sub>d</sub>.

### 3 The VIKOR Method

The VIKOR method is a technique to be implemented within MCDM problem. it was introduced as a multi attribute method to solve a discrete decision making problem with different units and conflicting criteria. This method is used for ranking and selecting from several alternatives. The VIKOR method determines compromise solution for a problem with conflicting criteria, which can help the decision makers to find a final decision.

Each alternative is evaluated according to each criterion function; the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. Consider  $m$  alternatives  $A_1, A_2, \dots, A_m$ . For alternative  $A_i$ , the rating of the  $j$ th aspect is denoted by  $f_{ij}$ . It means  $f_{ij}$  is the value of  $j$ th criterion function for the  $i$ th alternative ;  $n$  is the number of criteria. The VIKOR method is started with the equation (4):

$$L_{pi} = \left\{ \sum_{j=1}^n [(f_j^* - f_{ij}) / (f_j^* - f_j^-)]^p \right\}^{1/p} \quad 1 \leq i=1, \dots, m \quad (4)$$

In the VIKOR method  $S_i$  And formulate ranking measure. The solution obtained by min  $S_i$  is with a maximum group utility ("majority" rule), and the solution obtained by min is with  $R_i$  a minimum individual regret of the "opponent". Steps of the compromise ranking algorithm of the VIKOR method are as follows:

Step1: Find the best  $f_j$  and the worst  $f_j$  values of all criterion functions  $j=1, 2, \dots, n$ . Then, find  $f_j^*$  and  $f_j^-$  as follows:

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij}$$

Step2: Compute the values  $S_i$  and  $R_i$ ;  $i=1, 2, \dots, m$ ;

$$S_i = \sum_{j=1}^n w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \quad (5)$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \quad (6)$$

Where  $w_j$  are the weights of criteria, expressing their relative importance.

Step3: Compute  $Q_i$ ;  $i = 1, 2, \dots, m$ , by the equation (7) as below:

$$Q_i = v(S_i - S^*) / (S^- - S^*) + (1-v)(R_i - R^*) / (R^- - R^*) \quad (7)$$

Where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i,$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i,$$

$v$  is introduced as weight of the strategy of “the majority of criteria” (or “the maximum group utility”), here suppose that  $v = 0.5$ .

Step4: sort the alternatives by the values  $S$ ,  $R$  and  $Q$  in decreasing order. Now we have three ranking lists.

Step5: Propose as a compromise solution the alternative  $A'$ , which is ranked the best by the measure  $Q$  (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage:

$$Q(A'') - Q(A') \geq 1/(m-1) \quad (8)$$

where  $A''$  is the second alternative in the ranking list by  $Q$ ; that  $m$  is the number of alternatives.

Consider three alternatives with four criteria in table (2):

**Table 2** Three alternatives with four criteria

|       | $C_1^-$ | $C_2^+$ | $C_3^+$ | $C_4^+$ |
|-------|---------|---------|---------|---------|
| $A_1$ | 5       | 8       | 13      | 4       |
| $A_2$ | 4       | 10      | 9       | 2       |
| $A_3$ | 8       | 12      | 6       | 3       |

At first, we scale the data in table (2) and make table (3):

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

**Table 3** Scaled data in table (2)

|       | $C_1^-$ | $C_2^+$ | $C_3^+$ | $C_4^+$ |
|-------|---------|---------|---------|---------|
| $A_1$ | 0.488   | 0.456   | 0.769   | 0.743   |
| $A_2$ | 0.390   | 0.570   | 0.532   | 0.371   |
| $A_3$ | 0.781   | 0.684   | 0.355   | 0.557   |

and then we find  $f_j^* = \max_i f_{ij}$ ,  $f_j^- = \min_i f_{ij}$  in table (3) and make table (4):

**Table 4**  $f_j^*, f_j^-$

|         | $C_1^-$ | $C_2^+$ | $C_3^+$ | $C_4^+$ |
|---------|---------|---------|---------|---------|
| $f_j^*$ | 0.781   | 0.684   | 0.769   | 0.743   |
| $f_j^-$ | 0.390   | 0.456   | 0.532   | 0.371   |

The we use equations (5), (6) and (7) and make table (5) with  $w_j = 1$  for  $j = 1, 2, 3, 4$  and  $v = 0.5$ :

**Table 5** Rank by Q,S and R

| Q     |       | S    |       | R     |       |
|-------|-------|------|-------|-------|-------|
| 0     | $A_1$ | 0.32 | $A_1$ | 0.299 | $A_1$ |
| 0.655 | $A_3$ | 0.47 | $A_3$ | 0.305 | $A_2$ |
| 0.855 | $A_2$ | 0.81 | $A_2$ | 0.336 | $A_3$ |

### C2. Acceptable stability in decision making:

Alternative  $A'$  must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be “voting by majority rule” (when  $v > 0.5$  is needed), or “by consensus”  $v = 0.5$ , or “with veto” ( $v < 0.5$ ). Here,  $v$  is the weight of the decision making strategy “the majority of criteria” (or “the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- \_ Alternatives  $A'$  and  $A''$  if only condition C2 is not satisfied, or
- \_ Alternatives  $A'$ ,  $A''$ , ...,  $A^{(M)}$  if condition C1 is not satisfied;  $A^{(M)}$  is determined by the relation  $Q(A^{(M)}) - Q(A') < DQ$  for Maximum M (the positions of these alternatives are “in closeness”).

The decision maker should accept compromise solution because DM provides a maximum utility of the majority (represented by min S), and a minimum individual regret of the opponent (represented by min R). S and R are integrated into Q for compromise solution, the base for an agreement established by mutual concessions.

## 4 Problem Definition and the Proposed Method

Hafezolkotob et al. [40] have studied ROBUST DEA with different scenarios by proposing a case study from Electric Energy Distribution Company. These data are used for presenting the method. Firstly, the proposed method in this article which is a combination of data enveloping analysis and VIKOR is presented.

### 4.1 Method

Suppose that we have  $n$  DMUs and  $t$  scenarios for the inputs and outputs in which the occurrence probability of each scenario is specified. Consider  $p_j$  ( $j=1,2,\dots, t$ ) to be the occurrence probability of  $j^{\text{th}}$  scenario for each DMU, it is clear that  $\sum_{j=1}^t p_j = 1$ .

Using the combination of DEA and VIKOR, now we want to do the ranking of these units with specified probability of different scenarios occurrence. In the table (1), consider  $A_i$ s as the DMUs and  $C_j$ s as the  $n^{\text{th}}$  scenario for each DMU. Then,  $f_{ij}$  can be the efficiency score of the  $i^{\text{th}}$  decision making unit for the  $j^{\text{th}}$  scenario in the table (1). Therefore,  $f_{ij}$  can be easily obtained by using the model (3) for all the DMUs and consequently the table (2) is completed. The decided weighs for the VIKOR method in the table (1) which are  $w_j$  are considered as the occurrence probability of each scenario which is  $p_j$ . Therefore, the table (6) is compiled as below:

**Table 6** Efficiency score of DMUs with n scenarios

|         | $S_1$    | $S_2$    | ... | $S_n$    |
|---------|----------|----------|-----|----------|
| $DMU_1$ | $f_{11}$ | $f_{12}$ | ... | $f_{1n}$ |
| $DMU_2$ | $f_{21}$ | $f_{22}$ | ... | $f_{2n}$ |
| ...     | ...      | ...      | ... | ...      |
| $DMU_m$ | $f_{m1}$ | $f_{m2}$ | ... | $f_{mn}$ |

$$P=[p_1, p_2, \dots, p_n]$$

In the table (6),  $S_j$  ( $j=1, \dots, n$ ) is the available scenarios for each decision making unit. Now we can rank the decision making units by using the VIKOR method by the table (6).

We could rank the decision making units using the TOPSIS method. However, considering the article written by Opricovic, Tzeng [41], we decided to do the ranking using the VIKOR method.

## 4.2 Numerical example

To present the method DEA/VIKOR, a numerical example is given here. Hafezolkotob et al. [40] studied their proposed method named ROBUST DEA by providing a numerical example on electric energy distribution units in Iran with three different scenarios including pessimistic, probable and optimistic each of which has the occurrence probability of 0.25, 0.5 and 0.25 respectively. That problem and its data have been applied in this article. However, the DEA/VIKOR method has been used for ranking them.

There are 38 DMUs in this example in which the inputs are the number of workers, the length of network and the capacity of transformers; and outputs are total selling of power and the number of customers.

We achieved the efficiency scores of the decision making units by applying the model (3) in the table (7) for 38 decision making units which have three scenarios.

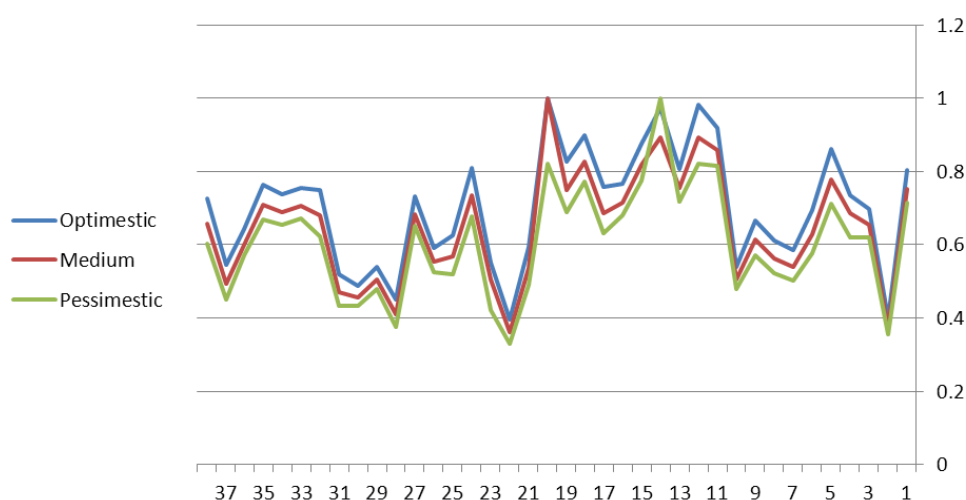
**Table 7** Efficiency score of DMUs with 3 scenarios

| DMUs  | $S_1$  | $S_2$  | $S_3$  |
|-------|--------|--------|--------|
| DMU1  | 0.8038 | 0.7512 | 0.7137 |
| DMU2  | 0.3999 | 0.3738 | 0.3551 |
| DMU3  | 0.6989 | 0.6532 | 0.6205 |
| DMU4  | 0.7349 | 0.6868 | 0.6205 |
| DMU5  | 0.861  | 0.7794 | 0.7133 |
| DMU6  | 0.6945 | 0.6296 | 0.5773 |
| DMU7  | 0.5848 | 0.5385 | 0.5029 |
| DMU8  | 0.6123 | 0.5621 | 0.523  |
| DMU9  | 0.6671 | 0.6131 | 0.5713 |
| DMU10 | 0.5403 | 0.5049 | 0.4797 |
| DMU11 | 0.919  | 0.8589 | 0.8159 |
| DMU12 | 0.9813 | 0.8924 | 0.8212 |
| DMU13 | 0.8075 | 0.7547 | 0.717  |
| DMU14 | 0.9724 | 0.8923 | 1      |
| DMU15 | 0.8744 | 0.8172 | 0.7763 |
| DMU16 | 0.7654 | 0.7153 | 0.6795 |
| DMU17 | 0.7573 | 0.687  | 0.6305 |
| DMU18 | 0.8975 | 0.826  | 0.7711 |

| DMUs  | $S_1$  | $S_2$  | $S_3$  |
|-------|--------|--------|--------|
| DMU19 | 0.826  | 0.7493 | 0.6876 |
| DMU20 | 1      | 1      | 0.8198 |
| DMU21 | 0.5943 | 0.5373 | 0.491  |
| DMU22 | 0.397  | 0.3601 | 0.3303 |
| DMU23 | 0.5498 | 0.5054 | 0.4212 |
| DMU24 | 0.8095 | 0.7359 | 0.6769 |
| DMU25 | 0.6264 | 0.5678 | 0.5206 |
| DMU26 | 0.5917 | 0.553  | 0.5253 |
| DMU27 | 0.7326 | 0.6846 | 0.6504 |
| DMU28 | 0.451  | 0.4096 | 0.3764 |
| DMU29 | 0.5398 | 0.5045 | 0.4793 |
| DMU30 | 0.4889 | 0.457  | 0.4341 |
| DMU31 | 0.5183 | 0.4706 | 0.4323 |
| DMU32 | 0.7507 | 0.68   | 0.6228 |
| DMU33 | 0.7557 | 0.7063 | 0.671  |
| DMU34 | 0.7367 | 0.6885 | 0.6541 |
| DMU35 | 0.7625 | 0.7085 | 0.6688 |
| DMU36 | 0.6421 | 0.6001 | 0.5701 |
| DMU37 | 0.5447 | 0.4929 | 0.451  |
| DMU38 | 0.7259 | 0.6582 | 0.6037 |

We complete the table (7) after achieving the efficiency scores of all the decision making units for different scenarios. Left to right, the first column of the data available is for the efficiency scores of the pessimistic scenario, the second is for probable scenario and the third column is for the optimistic scenario each of which has the occurrence probability of 0.25, 0.5 and 0.25 respectively.

Efficiency graphs can be drawn as below for DMUs based on their achieved efficiency score.



**Fig. 1** Efficiency graphs for DMUs with 3 scenarios

Regarding the figure (1) and the efficiency function of each scenario, we cannot achieve an appropriate ranking based on their efficiency score. As seen in the efficiency graph in the



figure (1), the optimistic graph is not always the best and the pessimistic mode is not always the worst. Similarly, the medium mode is not always placed between their efficiency graphs.

If we calculate the weighted average of efficiency graphs and consider the weights to be the occurrence probability of each scenario, we have the figure (2).

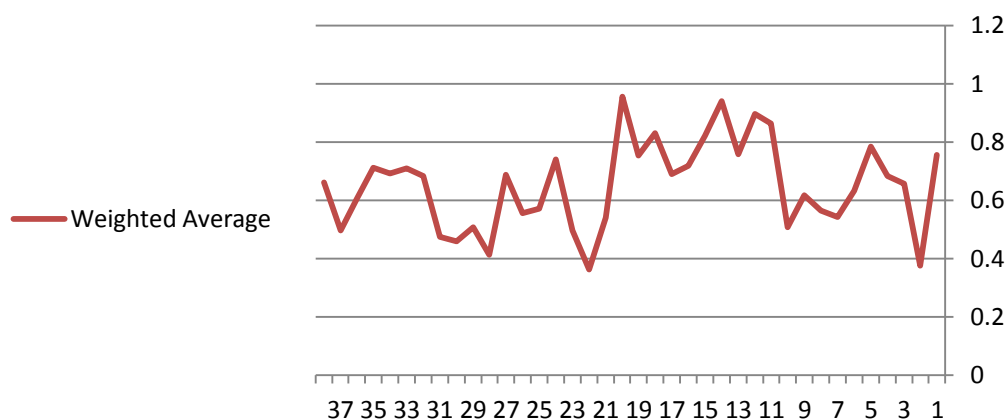


Fig.2 weighted average graph

Regarding the weight of the middle scenario, after calculating the weighted average, efficiency graph is much more similar to medium mode. In other words, optimistic and pessimistic modes are not used at all in calculating the efficiency score. Therefore, the weighted average method is not an appropriate method for calculating the efficiency score of decision making units with several scenarios.

Now we want to achieve the ranking of the decision making units with different scenarios and specified occurrence probability by using the proposed method in 4-1 and a combination of the DEA and VIKOR methods. As mentioned before, occurrence probability of the scenarios can be considered as weights in VIKOR method. Applying the VIKOR method for the data in table (7) and make the table (8):

Table 8 Q, S, R by VIKOR

| DMU   | $Q_i$    | $S_i$       | $R_i$       |
|-------|----------|-------------|-------------|
| DMU1  | 0.31595  | 0.382624873 | 0.194405376 |
| DMU2  | 0.976285 | 0.978835004 | 0.489295202 |
| DMU3  | 0.487441 | 0.537481914 | 0.270979841 |
| DMU4  | 0.435031 | 0.496302439 | 0.244725738 |
| DMU5  | 0.266045 | 0.337024741 | 0.172370683 |
| DMU6  | 0.528256 | 0.573873132 | 0.289420222 |
| DMU7  | 0.687931 | 0.718310688 | 0.360603219 |
| DMU8  | 0.646605 | 0.68096562  | 0.342162838 |
| DMU9  | 0.557354 | 0.600365447 | 0.302312861 |
| DMU10 | 0.746873 | 0.771674804 | 0.386857321 |
| DMU11 | 0.127548 | 0.212558494 | 0.110251602 |
| DMU12 | 0.068365 | 0.158574843 | 0.084075637 |
| DMU13 | 0.309841 | 0.377124179 | 0.191670574 |
| DMU14 | 0.034695 | 0.09559656  | 0.084153774 |
| DMU15 | 0.2005   | 0.278415327 | 0.142834818 |
| DMU16 | 0.378777 | 0.439363439 | 0.222456634 |

| DMU   | $Q_i$    | $S_i$       | $R_i$       |
|-------|----------|-------------|-------------|
| DMU17 | 0.427787 | 0.483126251 | 0.244569464 |
| DMU18 | 0.184776 | 0.263903306 | 0.135958744 |
| DMU19 | 0.31875  | 0.384648668 | 0.195889983 |
| DMU20 | 0        | 0.067268926 | 0.067268926 |
| DMU21 | 0.689787 | 0.719751982 | 0.361540866 |
| DMU22 | 1        | 1           | 0.5         |
| DMU23 | 0.755807 | 0.789183614 | 0.386466635 |
| DMU24 | 0.342269 | 0.405954176 | 0.206360369 |
| DMU25 | 0.636418 | 0.671561951 | 0.337709017 |
| DMU26 | 0.662751 | 0.695758143 | 0.349273324 |
| DMU27 | 0.432466 | 0.487813309 | 0.246444757 |
| DMU28 | 0.913349 | 0.921724824 | 0.461322082 |
| DMU29 | 0.747593 | 0.77234397  | 0.38716987  |
| DMU30 | 0.830731 | 0.84743519  | 0.424285045 |
| DMU31 | 0.806582 | 0.825291418 | 0.413658384 |
| DMU32 | 0.440046 | 0.494206595 | 0.250039069 |
| DMU33 | 0.394529 | 0.453590409 | 0.229488983 |
| DMU34 | 0.42566  | 0.481684908 | 0.243397406 |
| DMU35 | 0.39055  | 0.449873417 | 0.227769964 |
| DMU36 | 0.580332 | 0.621336089 | 0.312470699 |
| DMU37 | 0.767499 | 0.789940809 | 0.396233787 |
| DMU38 | 0.478193 | 0.528652489 | 0.26707298  |

**Table 9** Rank by Q, S, R

| Rank | Q      | S      | R      |
|------|--------|--------|--------|
| 1    | DMU 20 | DMU 20 | DMU 20 |
| 2    | DMU 14 | DMU 14 | DMU 12 |
| 3    | DMU 12 | DMU 12 | DMU 14 |
| 4    | DMU 11 | DMU 11 | DMU 11 |
| 5    | DMU 18 | DMU 18 | DMU 18 |
| 6    | DMU 15 | DMU 15 | DMU 15 |
| 7    | DMU 5  | DMU 5  | DMU 5  |
| 8    | DMU 13 | DMU 13 | DMU 13 |
| 9    | DMU 1  | DMU 1  | DMU 1  |
| 10   | DMU 19 | DMU 19 | DMU 19 |
| 11   | DMU 24 | DMU 24 | DMU 24 |
| 12   | DMU 16 | DMU 16 | DMU 16 |
| 13   | DMU 35 | DMU 35 | DMU 35 |
| 14   | DMU 33 | DMU 33 | DMU 33 |
| 15   | DMU 34 | DMU 34 | DMU 34 |
| 16   | DMU 17 | DMU 17 | DMU 17 |
| 17   | DMU 27 | DMU 27 | DMU 4  |
| 18   | DMU 4  | DMU 32 | DMU 27 |
| 19   | DMU 32 | DMU 4  | DMU 32 |
| 20   | DMU 3  | DMU 3  | DMU 38 |
| 21   | DMU 38 | DMU 38 | DMU 3  |

| Rank | Q      | S      | R      |
|------|--------|--------|--------|
| 22   | DMU 6  | DMU 6  | DMU 6  |
| 23   | DMU 9  | DMU 9  | DMU 9  |
| 24   | DMU 36 | DMU 36 | DMU 36 |
| 25   | DMU 25 | DMU 25 | DMU 25 |
| 26   | DMU 8  | DMU 8  | DMU 8  |
| 27   | DMU 26 | DMU 26 | DMU 26 |
| 28   | DMU 7  | DMU 7  | DMU 7  |
| 29   | DMU 21 | DMU 21 | DMU 21 |
| 30   | DMU 10 | DMU 10 | DMU 23 |
| 31   | DMU 29 | DMU 29 | DMU 10 |
| 32   | DMU 23 | DMU 23 | DMU 29 |
| 33   | DMU 37 | DMU 37 | DMU 37 |
| 34   | DMU 31 | DMU 31 | DMU 31 |
| 35   | DMU 30 | DMU 30 | DMU 30 |
| 36   | DMU 28 | DMU 28 | DMU 28 |
| 37   | DMU 2  | DMU 2  | DMU 2  |
| 38   | DMU 22 | DMU 22 | DMU 22 |

**Table 10** Ranked by VIKOR method

| Rank | DMU                                           |
|------|-----------------------------------------------|
| 1    | DMU20                                         |
| 2    | DMU14 - DMU12                                 |
| 3    | DMU11                                         |
| 4    | DMU18 - DMU15                                 |
| 5    | DMU5                                          |
| 6    | DMU13 - DMU1 - DMU24 - DMU19                  |
| 7    | DMU16 - DMU35 - DMU33                         |
| 8    | DMU34 - DMU17 - DMU 27 - DMU 4 - DMU32        |
| 9    | DMU3 - DMU38 - DMU6                           |
| 10   | DMU9 - DMU36                                  |
| 11   | DMU25 - DMU8 - DMU26 - DMU7 - DMU21           |
| 12   | DMU10 - DMU29 - DMU23 - DMU37 - DMU31 - DMU30 |
| 13   | DMU28 - DMU2 - DMU22                          |

According to the article written by Opricovic, Tzeng [41], it is decided to do the ranking by using the VIKOR method rather than TOPSIS method after achieving the efficiency score of each decision making unit with different scenarios.

## 5 Conclusions

Not so much research has been conducted on data with different scenarios which have specified occurrence probability and the proposed methods have difficult models for solving the problem and ranking the decision making units. Furthermore, infeasibility is probable in the proposed models. These models ignore the infeasibility supposition in order to rank which

is not a true criterion for ranking the decision making units. In this research, we believe that using the data enveloping analysis and Vikor method, we have proposed a method for ranking the units which can rank the decision making units with data which have different scenarios without suffering from the difficulties of the previously proposed models

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