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Performance evaluation of Iranian electricity distribution units with stochastic Data Envelopment Analysis

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Abstract Performance evaluation of electricity distribution units is an important issue between researchers and regulators. Classic Data Envelopment Analysis models with deterministic data have been used by many authors to measure efficiency of power distribution units in different countries. However, Data Envelopment Analysis with stochastic data are rarely used to measure efficiency of distribution companies. In this paper, input oriented model in stochastic Data Envelopment Analysis is used to evaluate power distribution units in Iran. In addition, variable returns to scale super efficiency model is used in stochastic Data Envelopment Analysis to rank stochastic efficient units. Deterministic equivalent of each stochastic model, which can be converted to a quadratic program, is applied to obtain numerical results with stochastic data. Under fairly general conditions, this model equivalent program is replaced by a linear programming problem. 38 Iranian electricity distribution units have been considered in this study for which computational results of both classic and stochastic approaches are obtained. It is shown that, on average, with considering deterministic (stochastic) data at most 89(94) percent of the resources for Iranian power distributions units is needed to make them technically efficient.

Key words: Stochastic data; efficiency; Power distribution units.

1 Preliminaries

In the last decades, performance evaluation of electricity distribution units has been considered as an important issue between researchers and regulators. Data Envelopment Analysis (DEA) which is based on a mathematical programming approach is an important methodology for measuring efficiency of decision making units (DMUs). This methodology has its strengths and limitations. In this methodology, the frontier is defined by the most efficient DMUs of the sample. Mathematically, DEA is introduced as a high reliability

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analysis tool and have been largely used for studies in the electrical filed. Fore example, Goto and Tsutsui [1] used the Data Envelopment Analysis (DEA) model to measure cost efficiency and technical efficiency between Japanese and US electricity utilities. They showed that Japanese utilities were more efficient than the US ones in terms of technical, allocative and scale efficiency. Resende [2] used non-parametric input-output DEA model for evaluating Brazilian electricity distribution firms. Jamasb and Pollit [3] compared 63 regional electricity distribution utilities for six European countries. To calculate efficiency and to consider the effects of the choice of variables and methods, they used ten DEA, corrected ordinary least square (COLS), and stochastic frontier analysis (SFA) models. Estache et al. [4] applied DEA and econometric methods for performance assessment and ranking of South American electricity units. Giannakis et al. [5] applied the DEA model to study service quality of UK electricity distribution utilities. A researcher is 2001 applied the DEA approach to measure technical efficiency of Chinese thermal power generation based on cross-sectional data for 1995 and 1996. Their results showed that municipalities and provinces along the eastern coast of China and those with rich supplies of coal achieved the highest level of technical efficiency. In the second stage regression analysis, they found that fuel efficiency and the capacity factor significantly affect technical efficiency. See, also, Sanhueza et al. [6], and Pollit [7]. Furthermore, Azadeh et al. [8] proposed an adaptive neural network algorithm to performance assessment of electric power generations. They applied the proposed approach on a real data set for measuring 19 power generations in Iran. Azadeh et al. [9] applied DEA and principle component analysis (PCA) for performance assessment and optimization of electricity distribution units in Iran. Moreover, Sadjadi and Omrani [10] applied Data Envelopment Analysis with uncertain data on Iranian electricity distribution companies. They proposed a DEA method with consideration of the uncertainty on output parameters. As some authors argue, see for example Coelli [11], in many studies the researchers have to select input oriented models because DMUs have particular orders to fill, e. g. power distributions, and hence the input quantities appear to be the primary decision variables.

The Data Envelopment Analysis makes no assumption about the functional form of the frontier. Instead, it specifies certain assumptions about the underlying technology that in combination with the data set allows the construction of the production set. For instance, the DEA frontier is very sensitive to the presence of the outliers and statistical noise which indicates that the frontier derived from DEA analysis may be warped if the data are contaminated by statistical noise, see Bauer [12]. To incorporate possible uncertainty in inputs and/or outputs, stochastic formulation of the original DEA models were introduced in the literature. See, for example, Olesen and Petersen [13], Cooper et al. [14], Li [15], Cooper et al. [16], Cooper et al. [17], Cooper et al. [18], Land et al. [19], Huang and Li [20]. See, also, Kall [21] for discussions on stochastic linear programming programs. More recent applications of stochastic DEA are described as follows. Odeck [22] pointed out that while DEA has many advantages such as modeling multi output and multi input technologies even in the absence of price data, the technique has one particular draw back. The drawback is that the estimates from convention DEA analysis offer no information on estimates uncertainty. Therefore, he provided an alternative and complementary approach to performance assessment as well as to the determinants of that performance. He contributes to the literature on efficiency and productivity measurement in the agricultural sector by employing Malmquist indices to measure productivity, DEA to measure efficiency and bootstrapping to ascertain confidence intervals for the estimators. He found that the mean potential for input saving among Norwegian grain producers has been approximately 11%. Kao and Liu [23] used stochastic Data Envelopment Analysis in measuring the efficiency of Taiwan commercial banks. The input/output data of their study was obtained from the Taiwan Stock Exchange Corporation (TSEC) which only contains annual data for each bank. Using data of five years as multiple observations for each bank, their average is used as the constant observation for calculating the efficiency. In their study an assumption that the data distributions in five years are the same has been made. They noted that the conventional DEA approach categorizes the DMUS as efficient and inefficient ones, on the other hand, the stochastic DEA approach does not give a sharp categorization. Therefore, many DMUS may not always be efficient, yet they have the chance to be recognized as efficient. The probabilistic type of categorization is psychologically more acceptable to the DMUS being evaluated, because unlike the classical 0-1 type categorization, it gives a DMU some chance of being classified as efficient. Khodabakhshi and Asgharian [24] applied stochastic input relaxation model on data of textile industry of China to evaluate technical efficiency with considering maximum possible output. They elaborated importance of their model to management of resources for textile industry. Khodabakhshi [25] applied stochastic input-output orientation model on data of software companies to obtain most productive scale size companies. He found that SNIC, VENQ, and Adobe can be used as benchmark for other research oriented companies to improve their performances. Furthermore, Khodabakhshi et al. [25] applied stochastic input oriented super efficiency model under constant returns to scale assumption to evaluate efficiency of chief executive officers of top US public banks and thrifts. See also Khodabakhshi et al. [26]. Alternatively, other authors proposed fuzzy mathematical programming approaches of Data Envelopment Analysis to deal with variations in data see, for example, Kao and Liu [27], Liu [28], Leon et al. [29], and Lertworasirikul et al. [30]. Here, we are interested in stochastic variations in input-output data. In this paper, first, the classic input orientation model introduced in Banker et al. [31] is used to evaluate electricity distribution units in Iran. The super efficiency model introduced in Anderson, Petersen [32] is also used to rank efficient units. One may refer to, for example, Adler et al. [33], Mehrabian et al. [34], Martic and Savic [35], Tone [36], Li et al. [37], and Khodabakhshi [38] to see further research on super efficiency in DEA. Next, efficiency of power distribution units with considering stochastic data is measured. Then, we extend variable returns to scale input-oriented super-efficiency model, allowing deterministic inputs and outputs to be stochastic. The stochastic super efficiency model is used to rank stochastic efficient units. To obtain numerical results with stochastic data, a deterministic equivalent of each stochastic model is used which can be converted to a quadratic program. Computational results of the classic and stochastic approaches are compared, too.

The rest of the paper is organized as follows. Input oriented BCC model, and Input oriented super-efficiency model in classic DEA are introduced in subsection 2. Subsection 2, also, includes stochastic input orientation model and its deterministic equivalent. Moreover, the input oriented super efficiency model is developed in stochastic data envelopment analysis, and its deterministic equivalent is also obtained. Furthermore, it is shown that the deterministic equivalent of the stochastic super efficiency model can be converted to a quadratic program. Section 2 applies the classic and stochastic approaches on data of Iranian electricity distribution companies. Section 3 concludes the paper.

The following subsections contain some classic DEA models and their extensions in stochastic DEA. Specifically, variable returns to scale input oriented super efficiency model is developed in stochastic DEA.

1.1 Classic DEA

It is assumed that there are n homogeneous DMUs such that all the DMUs use m inputs x_{ij} (i=1,...,m) to produce s outputs y_{rj} (r=1,...,s) . We also assume that x_{ij} and y_{rj} are non-negative deterministic elements.

1.1.1 Classic DEA

One of the basic models used to evaluate DMUs efficiency is the input-oriented BCC model introduced by Banker, Charnes, and Cooper [31]. This model is as follows.

$$\begin{aligned}
&\text{Min}\,\theta_o+\varepsilon\left(\sum_{i=1}^{m}s_i^-+\sum_{r=1}^{s}s_r^+\right)\\
&S.t.\quad \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=\theta x_{io}\quad ;\quad i=1,\dots,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-y_{ro}-s_r^+=0\quad ;\quad r=1,\dots,s\\
&\sum_{j=1}^{n}\lambda_j=1 \qquad\qquad (1)\\
&\lambda_j\ge 0,s_i^-\ge 0,\ s_r^+\ge 0\quad ;\quad j=1,\dots,n\ ,i=1,\dots,m,\ r=1,\dots,s
\end{aligned}$$

Definition 1. (Efficiency according to model(1)): DMU0 is efficient when in optimal solution(s)

i) $\theta_o^*=1$

ii) $s_i^{-*}=s_r^{+*}=0$, i=1,…,m, r=1,…,s.

One may refer to, for example, Charnes et al. [39], Cooper et al. [40], Jahanshahloo and Khodabakhshi [41,42], Thanassoulis [43], Cooper et al. [44] and Khodabakhshi [45] to see further classic models in DEA.

1.1.2 Super efficiency model

Excluding the column vector correspond to DMUo from the LP coefficients matrix of model (1) input oriented super-efficiency model introduced by Andersen and Petersen [32] is defined as follows:

$$\begin{aligned}
&\text{Min}\quad \theta_o^s\\
&Subject\ \ to\quad \sum_{j=1,j\ne o}^{n}\lambda_j x_{ij}\le\theta x_{io}\quad ;\quad i=1\,,\dots,m\\
&\sum_{j=1,j\ne o}^{n}\lambda_j y_{rj}\ge y_{ro}\quad ;\quad r=1\,,\dots,s\\
&\sum_{j=1,j\ne o}^{n}\lambda_j=1 \qquad\qquad (2)\\
&\lambda_j\ge 0,\ j=1,\dots,n,\ j\ne o
\end{aligned}$$

Although the model is used to rank extreme DEA efficient DMUs obtained by the BCC model, it can also be used to evaluate efficient units directly. Therefore, without solving the BCC model, one can rank efficient DMUs by simply solving the super-efficiency model. Efficient DMUs have super-efficiency score greater than or equal to 1, while inefficient DMUs have super efficiency score less than 1. To explain this point more precisely, if the optimal objective value of the super-efficiency model is greater than 1, DMUo that is DEA efficient in the BCC model is super efficient. Otherwise, DMUo is not super efficient. Thus, one can solve the super-efficiency model for ranking efficient units without solving the BCC model. The super-efficiency scores of the DMUs obtained by the above super-efficiency model can then be ranked in a descending order.

1.2 Stochastic DEA

In what follows, an approach which allows for the possible presence of stochastic variability in the data is introduced. Following Cooper et al. [17] and Khodabakhshi and Asgharian [24], let $\tilde{x}_j=(\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})^t$, $\tilde{y}_j=(\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})^t$ be random input and output related to DMU_j (j=1, …, n). Let also $x_j=(x_{1j},\ldots, x_{mj})^t$, $y_j=(y_{1j},\ldots, y_{sj})^t$ show the corresponding vectors of expected values of inputs and outputs for DMUj. Suppose that all input and output components are jointly normally distributed. Following Cooper et al. [17], the corresponding stochastic version of Model (1), including slack variables, is:

$$\text{Min} \quad \theta_o + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$s.t. \qquad p\left\{ \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} + s_i^- \le \theta \tilde{x}_{io} \right\} \ge 1-\alpha \qquad\qquad i=1,\ldots,m$$

$$p\left\{\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} - \tilde{y}_{ro} \ge s_r^+ \right\} \ge 1-\alpha \qquad\qquad r=1,\ldots,s$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{3}$$

$$s_i^-, \lambda_j, s_r^+ \ge 0$$

where α, a predetermined value between 0 and 1, specifies the significance level, and P represents the probability measure.

Definition 2: (Stochastic efficiency according to model (3) DMUo is called stochastically efficient at significance level α if the following conditions are fulfilled.

i) $\theta_o^*=1$

ii)$s_i^{-*}=s_r^{+*}=0$; for all i and r

DMUo is called stochastically inefficient if it doesn't fulfill the conditions of Definition 2. In other words, if for an optimal solution $\theta_o^*<1$, or some of slacks are non zero, then DMUo is stochastically inefficient. In fact, if $\theta_o^*<1$, then all inputs for evaluating DMUo can be decreased to $\theta_o^* x_{io}$, (i=1 ,…, m) by using a convex combination of the other DMUs at the significance level α.

1.2.1 Stochastic input oriented super-efficiency model

Based on the previous assumptions, the stochastic version of the input oriented super-efficiency model can be defined as follows.

$$\begin{aligned}
&Min \quad \theta \\
&s.t. \quad p\left\{ \sum_{j=1, j\neq o}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io} \le 0 \right\} \ge 1-\alpha \qquad i=1,\dots,m \\
&\qquad p\left\{ \sum_{j=1, j\neq o}^{n} \lambda_j \tilde{y}_{rj} - \tilde{y}_{ro} \ge 0 \right\} \ge 1-\alpha \qquad r=1,\dots,s \\
&\qquad \sum_{j=1, j\neq o}^{n} \lambda_j = 1 \\
&\qquad \lambda_j \ge 0
\end{aligned} \tag{4}$$

DMUo is stochastically super-efficient at significance level α if the optimal value of the objective function is greater than 1. Therefore, if $\theta_o^{s*}>1$ it means that even if DMUo consumes θ_o^{s*} percent of its current input it can remain efficient, hence the greater the θ_o^{s*}, the better the DMU. In what follows, the deterministic equivalent of the above stochastic super-efficiency model is obtained.

1.2.2 Deterministic equivalents

In this subsection, we exploit the Normality assumption to introduce a deterministic equivalent to model (4). It is assumed that x_{ij} and y_{rj} are the means of the input and output variables, which are, in application, observed values of the inputs and outputs. Following Khodabakhshi and Asgharian [24], the deterministic equivalent of (3) can be represented by the following quadratic program.

$$\begin{aligned}
&\text{Min} \quad \theta_o + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right) \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- - \phi^{-1}(\alpha) w_i^I = \theta_o x_{io}, i=1,\dots,m
\end{aligned}$$

$$y_{ro} - \sum_{j=1}^{n} \lambda_j x_{ij} + s_r^+ - \phi^{-1}(\alpha) w_r^0 = 0, r = 1,...,s$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (5)$$

$$(\omega_i^I)^2 = \sum_{j \neq 0} \sum_{k \neq 0} \lambda_j \lambda_k \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{ik}) + 2(\lambda_0 - \theta_o) \sum_{j \neq o} \lambda_j \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{i0}) + (\lambda_o - \theta_o)^2 \operatorname{var}(\tilde{x}_{io})$$

$$(\omega_r^0) = \sum_{j \neq 0} \sum_{k \neq 0} \lambda_k \lambda_j \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{rj}) + 2(\lambda_0 - 1) \sum_{k \neq o} \lambda_k \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{ro}) + (\lambda_0 - 1)^2 \operatorname{var}(\tilde{y}_{ro})$$

$$s_i^-, \lambda_j, s_i^+, \omega_i^I, \omega_r^o \geq 0$$

One can, therefore, obtain the optimal values θ_o, s_i^{-*} and s_r^{+*} by solving the quadratic program. Finally, in a similar fashion, the following deterministic equivalent for the stochastic input oriented super-efficiency model is obtained.

$$\text{Min} \quad \theta_o$$

$$s.t. \quad \sum_{j=1, j \neq o}^{n} \lambda_j x_{ij} + s_i^- - \phi^{-1}(\alpha) w_i^I = \theta_o^s x_{io}, i = 1,...,m$$

$$y_{ro} - \sum_{j \neq o, j=1}^{n} \lambda_j y_{rj} + s_r^+ - \phi^{-1}(\alpha) w_r^0 = 0, r = 1,...,s$$

$$\sum_{j \neq o, j=1}^{n} \lambda_j = 1 \quad (6)$$

$$(\omega_i^I)^2 = \sum_{j \neq 0} \sum_{k \neq 0} \lambda_j \lambda_k \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{ik}) - 2\theta_o^s \sum_{j \neq o} \lambda_j \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{i0}) + (\theta_o^s)^2 \operatorname{var}(\tilde{x}_{io})$$

$$(\omega_r^0)^2 = \sum_{j \neq 0} \sum_{k \neq 0} \lambda_k \lambda_j \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{rj}) - 2 \sum_{k \neq o} \lambda_k \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{ro}) + \operatorname{var}(\tilde{y}_{ro})$$

$$s_i^-, \lambda_j, s_i^+, \omega_i^I, \omega_r^o \geq 0$$

where ϕ is the cumulative distribution function (cdf) of a standard Normal random variable and ϕ^{-1} is its inverse. It is assumed that x_{ij} and y_{rj} are the means of the input and output variables, which are the observed values of the inputs and outputs in the following application.

2 Data and results

This study considers annual data of 38 Iranian electricity distribution companies observed in 2004. The Iranian electricity distribution units established in 1992, are public and act under the supervision of TAVANIR Company (Iran power, Generation, Transmission and Distribution Management Company). According to the extensive review in Jamasb and

Pollittt [46], the most frequently used inputs are operating costs, number of employees, transformer capacity and network length. The most widely useful outputs are also units of energy delivered, number of customers and size of service area. Note that the network length and transformers capacity are capital variables for electricity distribution units. The cost data usually are not available. The measurement units for the network length, transformers capacity and total electricity sales are Kilometer (KM), MVA and MWh, respectively. This study used network length, transformer capacity, and employee variables as inputs and total electricity sales and number of customers variables as outputs. Table 1 shows the data and summary statistics for the case study, see Sadjadi and Omrani [10].

Table 1 Data of electricity distribution units

	Labor	Net. Length	Cap. Transformers	Sales	No. Customers
1	540	8790	1106	1951	505.546
2	849	17680	1030	1450	479.527
3	1001	20118	1345	2191	650.487
4	405	10363	530	763	292.306
5	575	8771	1356	3153	630.812
6	1028	26399	2866	5218	768.629
7	290	7802	583	831	186.856
8	585	15032	1578	2563	409.273
9	573	13347	1217	1914	427.753
10	566	12498	1060	1253	335.4
11	664	12183	2368	4572	843.957
12	512	5191	1842	4664	721.002
13	530	8710	1968	3117	614.745
14	555	4829	1553	3356	753.741
15	650	7875	1706	3847	763.018
16	806	10571	2295	4140	835.5
17	324	5628	805	1655	278.532
18	752	11608	1696	3620	780.217
19	1439	47237	2605	6411	1057.901
20	524	5385	2297	6237	260.814
21	1348	19916	4081	7000	515.665
22	363	5429	571	692	110.375
23	344	10567	809	1236	234.129
24	292	9031	1003	2327	307.647
25	360	8363	768	1357	196.944
26	507	13230	1210	1583	420.976
27	398	12082	767	1097	324.524
28	241	5125	455	563	114.151
29	717	13480	2008	2822	534.869
30	759	25735	1970	3332	515.333
31	354	7522	1163	1826	200.817
32	997	31554	2145	4935	598.648
33	900	21665	1634	2515	777.329
34	773	18897	1877	2453	702.745
35	269	6224	645	868	269.786
36	407	10498	1005	1327	359.538
37	924	16510	1865	3091	304.896
38	596	12595	1022	2335	355.344
mean	624	13380	1494.842	2744	485.2561

From computational results of Table 2 efficiency score of power distribution companies 4, 5, 11, 12, 14, 16, 17, 19, 21, 24, 28, 33 and 35 are equal to unity. Therefore, these DMUs are efficient. Note that the slack values for these DMUs are all zero which we refrained from providing them in separate columns, therefore, they are strong efficient, i. e. they satisfy in both conditions (i) and (ii) of definition 1. From using resources point of view, these companies are technically efficient. In other words, they have no extra resources or wastes in their inputs. The rest of companies are inefficient. Inefficient companies obtained efficiency scores, ranging from a minimum of 0.6248, DMU 29, to 0.999, DMU 38, making them inefficient. For example, company 10 has efficiency score 0.675. It means that this company wastes (1-0.675) =0.325 percent of its current resources. In other words, 0.675 percent of resources for this company is enough to make it technically efficient. The inefficient DMUs are comparable using their efficiency scores which are less than one. However, efficient DMUs which have efficiency scores of unity are not comparable among themselves. Therefore, the super efficiency model is used to rank them. The results of super efficiency model and rank of DMUs are presented in columns 3 and 4, respectively. Based on the results of super efficiency model, company 20, with super efficiency score 2.8 is ranked the first. It means that, even if DMU 20 consumes 2.8 times of its current inputs, in comparison to other companies, it remains efficient. Note that the higher the super efficiency score, the better the DMUs. After DMU 20 the next three Companies are, DMU 21, 2.58, DMU 12, 1.85 and DMU 14, 1.39. At the bottom, the worst DMU is DMU 29 with efficiency score or super efficiency score 0.62485. In fact, DMU 29 with only 62.485 percent of its current inputs can be efficient, while 37.515 percent of its current inputs is wasted. Note that for inefficient DMUs super efficiency score is the same as the efficiency scores. After DMU 29, three next weak DMUs are DMU 37, 0.6739, DMU 10, 0.6749 and DMU 30, 0.69721. In overall, the mean technical efficiency score of power distribution units is 0.8965. In other words, in average, 10.35 percent of resources (inputs) for Iranian power distribution industry is wasted.

Table 2 Computational Results of classic models

.DMU No	Efficiency score	Super efficiency score	Rank
1	0.9069	---	23
2	0.9175	---	21
3	.0.9845	---	17
4	1	1.2950	7
5	1	1.0338	12
6	0.7221	---	33
7	.0.9523	---	19
8	0.7015	---	34
9	0.7626	---	31
10	0.6750	---	36
11	1	1.2576	8
12	1	1.8569	3
13	0.8581	---	26
14	1	1.3935	4
15	0.9861	---	16
16	1	1.0873	11
17	1	1.0197	13
18	0.9697	---	18
19	1	1.3150	5
20	1	2.8057	1
21	1	2.5899	2

.DMU No	Efficiency score	Super efficiency score	Rank
22	0.9399	---	20
23	0.8429	---	27
24	1	1.1774	9
25	0.9135	---	22
26	0.7463	--	32
27	0.8961	---	24
28	1	1.2971	6
29	0.6249	---	38
30	0.6972	---	35
31	0.8335	---	28
32	0.8923	---	25
33	1	1.0004	14
34	0.7672	---	30
35	1	1.1602	10
36	0.8041	---	29
37	0.6739	---	37
38	0.9990	---	15

Observed data of 38 companies for Iranian electricity distribution, also, is used to calculate stochastic efficiency and super efficiency scores for each company in 2004. To compute results for stochastic data, $\alpha \cong 0.45$ has been chosen for which $\phi^{-1}(\alpha) \cong$ -0.12. This rather large value of α is deliberately chosen to illustrate differences between the results based on the classic and the stochastic one. It is assumed that all DMUs have the same variance, but they can have different means. The variances for the outputs and the inputs can therefore be estimated by:

$$\text{Var}(\tilde{y}_r) = \frac{1}{37}\sum_{j=1}^{38}(y_{rj} - \bar{y}_r)^2, \quad \text{Var}(\tilde{x}_i) = \frac{1}{37}\sum_{j=1}^{38}(x_{ij} - \bar{x}_{i\ r)})^2,$$

where

$$\bar{y}_r = \frac{1}{38}\sum_{j=1}^{38} y_{rj} \quad \text{and} \quad \bar{x}_i = \frac{1}{38}\sum_{j=1}^{38} x_{ij}$$

and x_{ij} and y_{rj} are the observed values of inputs and outputs for DMU_j which we used as an estimate for the expected values of the stochastic inputs and outputs. It is, also, assumed that outputs and inputs for different DMUs are independent. This independence assumption then implies that Cov($\tilde{y}_{rk}, \tilde{y}_{rj}$)=0 and also Cov($\tilde{x}_{ij}, \tilde{x}_{ik}$)=0. Models (5) and (6) can be, therefore, converted to linear programs which can be solved using the simplex method. However, the stochastic results obtained from GAMS software are presented in Table 3. Computational results of stochastic efficiency and super efficiency models, their deterministic equivalents, are presented in Table 3. Columns 2, 3 and 4 represent efficiency scores, super efficiency scores and rank of DMUs, respectively. Based on the efficiency results, column 2, efficiency scores for DMUs 6, 8-10, 13, 26, 29-32, 34, 36, and 37 are less than one. Therefore, these DMUs are inefficient. Inefficient companies obtained efficiency scores, ranging from a minimum of 0.6861, DMU 29, to 0.9688, DMU 32, making them inefficient. The rest of DMUs which have efficiency score of unity are efficient. Optimal value of slack variables for efficient companies are, also, zero in stochastic model. To rank efficient units, the results of super efficiency model which are presented in column 3 of Table 3 is used. Based on the stochastic super efficiency results, again, DMU 20 with score 3.81 is ranked the first. Four top

DMUs in this evaluation are DMU 20, 3.817, DMU 21, 2.81, DMU 12, 2.7447 and DMU 14, 2.1588, respectively. Note that these DMUs are top with the classic and stochastic models. The worst DMU is, again, DMU 29 with score 0.6861. This DMU is the worst DMU by the deterministic model, too. At the bottom, after DMU 29 the next DMUs are, DMU 37, 0.7476, DMU 30, 0.7673, DMU 10, 0.7883 which have weak performances in this evaluation. The mean overall technical efficiency score of power distribution units is 0.9478. In other words by considering stochastic data, in average, 94.78 percent of resources (inputs) for Iranian power distribution companies is enough to make them technically efficient.

Table 3 Computational Results of stochastic models

.DMU No	Efficiency score	Super efficiency score	Rank
1	1	1.0529	24
2	1	1.0715	23
3	1	1.1046	21
4	1	1.6762	6
5	1	1.1643	17
6	0.8421	---	32
7	1	1.2248	13
8	0.7902	---	34
9	0.8762	---	30
10	0.7883	---	35
11	1	1.6840	5
12	1	2.7447	3
13	0.9479	---	28
14	1	2.1588	4
15	1	1.2099	14
16	1	1.6279	7
17	1	1.2423	12
18	1	1.1194	20
19	1	1.4057	9
20	1	3.8170	1
21	1	2.8190	2
22	1	1.1827	15
23	1	1.0191	25
24	1	1.3841	11
25	1	1.1258	18
26	0.8524	--	31
27	1	1.0834	22
28	1	1.5904	8
29	0.6861	---	38
30	0.7673	---	36
31	0.9657	---	27
32	0.9688	---	26
33	1	1.1216	19
34	0.8406	---	33
35	1	1.4022	10
36	0.9455	---	29
37	0.7476	---	37
38	1	1.1690	16

3 Conclusion

In this paper, Iranian electricity distribution companies are evaluated by classic and stochastic Data Envelopment Analysis. Input orientation BCC model introduced in Banker et al. [31] is used to measure technical efficiency of Iranian electricity distribution units in classic DEA. Furthermore, input oriented super efficiency model of Andersen and Peterson [32] is used to rank efficient units. Stochastic version of the input oriented model is also used to measure stochastic efficiency of power distribution companies. Moreover, variable returns to scale super Efficiency model is developed in stochastic DEA to rank stochastic efficient units. Numerical results obtained by the two approaches are compared, too. Finally, performance evaluation of power distribution units by considering fuzzy data can be suggested for further research.

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