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Voting and different ranking orders in DEA

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Abstract Data Envelopment Analysis (DEA) technique now widely use for efficiency evaluation of a set of Decision Making Units (DMUs). As regards of the necessity for ranking efficient units different DEA models presented each of which has advantages and rank efficient units from special aspects. Note that all the existing ranking models have disadvantages, as well and there is not a model in which all the benefits of different ranking model gathered. The aim of this paper is to provide a new ranking method which, to a great extent, consider advantages of various ranking models. In doing so voting technique, on basis of the obtained ranking order of other ranking methods, utilized. For clarity an application of this new method in banking system is provided.

Keywords: Data envelopment analysis, Ranking, Vote.

1 Introduction

Data Envelopment Analysis is now considered as a non-parametric technique which uses mathematical programming for the efficiency evaluation of a set of Decision Making Units (DMUs). As stated in literature various DEA models presented for raking efficient units. Efficient units are those located onto the production function frontier. The relative efficiency score of DMUs relates to the distance from the under evaluation unit from the frontier. The efficiency score of efficient units, those located onto the frontier, is equal to one. Thus these units can not be compared to each other any more. Therefore, introducing a secondary goal is necessary. Many researches have been done for introducing secondary goal in order to fully rank efficient units. Each of these methods uses different aspects and have priority over each other. Anderson and peterson (A.P) [1] proposed a method for ranking efficient DMUs based on the position of each eliminated efficient DMU in relation to its corresponding new Production Possibility Set (PPS). This model would be instable or infeasible in some cases as Thrall [2] showed in his paper. For overcoming this difficulty, Mehrabian *et al*. [3] introduced another method (MAJ) for ranking efficient DMUs. This method does not suffer from being nonstable but in some cases it would be infeasible. Thus, for overcoming mentioned difficulties of A.P. model, Jahanshahloo *et al*. [4] proposed Modified MAJ model in which the mentioned problems are fixed. Also, Sueyoshi [5] for overcoming the problem of

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instability in A.P model use the modified slacks-based model to rank efficient units. For avoiding mentioned difficulties in ranking efficient units Tone [6] presented a model called SBM. This models deals with slack variables and according to the idea of one-leave-out ranks units. Also, Li *et al* [7] proposed a super-efficiency model that does not have the suffering in previous methods. Sexton *et al*. [8] proposed another approach known as the cross-evaluation method, which can be utilized for ranking DMUs. Tohidi *et al*. [9] provided a method, in accordance to gradient line, for ranking efficient DMUs. As Tohidi *et al*. [9] discussed the advantage of this method is its stability and robustness. Jahanshahloo *et al*. [9] presented a method for ranking extreme efficient unit based on DEA technique. This method is based upon l1-norm distance of the under evaluation unit from new production possibility set. The great features of this model is that it is always feasible and stable. In their paper Rezai Balf *et al*. [10] provided a method, which has superiority over other existing methods, based on Tchebycheff Norm for ranking efficient units. Using the concept of common set of weights, Jahanshahloo *et al*. [11] presented a new model for determining the ranking order of units. Based upon the omission of these efficient DMUs from the reference set of the inefficient DMUs, Jahanshahloo *et al*. [12] proposed a new ranking system for extreme efficient DMUs. Liu and Peng [13] introduced common weights analysis (CWA) to determine the single most favorable common set of weights for DMUs on the DEA frontier in view of maximizing the groups efficiency score and they ranked DMUs based this idea. Soltanifar and Hosseinzadeh Lotfi [14] presented a paper considered voting analytic hierarchy process (VAHP) for ranking efficient units. Foroughi and Tamiz [15] in their paper introduce a model based on DEA technique for ranking candidates. The great feature of this model as the authors states is that it can rank both efficient and inefficient units. In this paper the aim is to provide a new ranking method based on the existing ranking models in literature. As each of the presented models have both advantages and disadvantages, thus here it is tried to get use of all these methods to present a new ranking method. In doing so a method presented for ranking efficient units based on voting technique while ranking order obtained from other ranking model, existed in DEA literature, utilized.

This paper unfolds as follows: at first some preliminaries about DEA technique will be reviewed. Then, in section 2 the new method for ranking efficient units considering different ranking models will be presented. Sections 4 and 5 give a numerical illustration of an application of the presented model in banking system and conclude the paper.

2 Priliminaries

2.1 Data Envelopment Analysis

Data envelopment (DEA) technique as a measurement tool widely used for evaluating a set of homogenous Decision Making Units (DMUs). This technique introduced by Charnes *et al*. [16] and then generalized by Banker *et al*. [17]. As an strong measurement tool this technique has been developed to a great extend. Each of the presented models in literature considers different aspects in performance evaluation. Thus, the obtained results can help managers to better making decision. One important issue in DEA technique is ranking efficient units since the efficiency score of efficient units are equal to 1 and therefore these units are not comparable to each other. Recognizing efficient units and ranking these units is of great importance since these units can help manages for setting target and better guiding the system. Assume there exist a set of n DMUs with m inputs and s outputs to be evaluated. It should be noted that in DEA assumptions it is assumed that the input and output vectors are all semipositive. Consider CCR model provided by Charnes *et al.* [16] in input orientation as follows:

$$
\begin{aligned}
& Min \quad Z_P = \theta - \varepsilon(1S^- + 1S^+) \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + S^- = \theta x_{io}, \qquad i = 1,\ldots,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} - S^+ = y_{ro}, \qquad r = 1,\ldots,s, \\
& \qquad \lambda_j \geq 0,\ S^- \geq 0, S^+ \geq 0, \qquad j = 1,\ldots,n.
\end{aligned} \tag{1}
$$

shows different form of technology, constant, variable, non-increasing and non decreasing returns to scale. The dual of the above model which is called multiplier form is as follows. In the following model v and u are the input and output weight vectors to be evaluated.
Considering variable returns to scale for of technology the dual model is:

$$
\begin{aligned}
& Max \quad \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1,\ldots,n, \\
& \qquad \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \qquad u_r \geq \varepsilon, \qquad r = 1,\ldots,s, \\
& \qquad v_i \geq \varepsilon, \qquad i = 1,\ldots,m.
\end{aligned} \tag{2}
$$

3 Proposed Method

There exist different ranking method in DEA literature. Each of these ranking methods rank units from different aspects thus has advantages and disadvantages. Note that until now there is no method which has all the ability of these method and a power for ranking all DMUs. Here considering voting technique and the model presented by Foroughi and Tamiz [15] a method for ranking efficient units will be presented.

Considering CCR model and identify efficient units. As stated in literature the quantity of θ^* for inefficient units is a criterion for ranking them but efficient units. Efficient units have efficiency score of 1 thus they can not be compared to each other any more according to this criterion. Thus a secondary goal is needed. Considering the ranking order obtained through ranking models we consider the vote that the ith candidate received from the jth voter. In this new method, for ranking efficient units, the ranking orders considered as votes that different ranking method give to efficient units. The ranking model presented by Foroughi and Tamiz [15] for a ranked voting system is as following. In this model d(., 0) is discrimination intensity function. Note that this function is nonnegative and non decreasing.

$$
\begin{aligned}
& Z^*_p = Min \ \sum_{i=1}^{n} \lambda_i - \varepsilon(\sum_{j=1}^{t} d_j S_j) \\
& s.t. \quad \sum_{i=1}^{n} \lambda_i r'_{ij} - S_j = \theta r'_{pj}, \qquad i = 1,...,m, \\
& \qquad \lambda_i \geq 0, \ S_j \geq 0, \qquad j = 1,...,n.
\end{aligned} \tag{3}
$$

where r'_{ij} is the vote that the ith candidate received from the jth voter. This model is as a two-phase problem. As Foroughi and Tamiz [15] noted, the optimal value of the second phase help to break the ties.

Now the weights using the results of the previous stage, those mentioned in Table 4, need to be derived. In doing so, the relative weight of each ranking model will be obtained by dividing the values in each column of Table 4 by the sum of the values in that column.

Now, considering the the ranking orders and corresponding weights for each ranking method and running model (3.3) a new ranking order for efficient units will be obtained. In this method ranking order acquired from other ranking models and also the measures obtained through solving these models thus it implicitly considerers the advantages of ranking methods.

4 Application

In this section, an empirical application in Commercial banks is performed. In doing so twenty Commercial banks of Iran are considered. The input-output data are listed in Table 1. In summary, the input and output sets are as follows. Also the result of CCR model as reviewed in previous sections, are listed in this table. As it can be seen seven units are efficient, DMUS 1,4,7,12,15, 17 and 20.

Inputs:
• Staff.
• Computer terminal.
• Space.
Outputs:
• Deposits.
• Loans Granted.
• Charge.

Table 1 Inputs, Outputs and efficiency scores

DMUp	I1	I2	I3	O1	O2	O3	CCR Efficiency
DMU1	0.95	0.7	0.155	0.19	0.521	0.293	1
DMU2	0.796	0.6	1	0.227	0.627	0.462	0.8333
DMU3	0.798	0.75	0.513	0.228	0.97	0.261	0.9911
DMU4	0.865	0.55	0.21	0.193	0.632	1	1
DMU5	0.815	0.85	0.268	0.233	0.722	0.246	0.8974
DMU6	0.842	0.65	0.5	0.207	0.603	0.569	0.7483
DMU7	0.719	0.6	0.35	0.182	0.9	0.716	1

DMUp	I1	I2	I3	O1	O2	O3	CCR Efficiency
DMU8	0.785	0.75	0.12	0.125	0.234	0.298	0.7978
DMU9	0.476	0.6	0.135	0.08	0.364	0.244	0.7877
DMU10	0.678	0.55	0.51	0.082	0.184	0.049	0.29
DMU11	0.711	1	0.305	0.212	0.318	0.403	0.6045
DMU12	0.811	0.65	0.255	0.123	0.923	0.628	1
DMU13	0.659	0.85	0.34	0.176	0.645	0.261	0.8166
DMU14	0.976	0.8	0.54	0.144	0.514	0.243	0.4693
DMU15	0.685	0.95	0.45	1	0.262	0.098	1
DMU16	0.613	0.9	0.525	0.115	0.402	0.464	0.639
DMU17	1	0.6	0.205	0.09	1	0.161	1
DMU18	0.634	0.65	0.235	0.059	0.349	0.068	0.4727
DMU19	0.372	0.7	0.238	0.039	0.19	0.111	0.4088
DMU20	0.583	0.55	0.5	0.11	0.615	0.764	1

Some of the important ranking methods in literature are listed in Table 2.

Table 2 Ranking Methods

Different ranking methods		
R.M1:	AP	[1]
R.M2:	MAJ	[3]
R.M3:	Modified MAJ	[12]
R.M4:	Change reference set	[4]
R.M5:	LJK	[7]
R.M6:	SBM	[6]
R.M7:	SA DEA	[5]
R.M8:	Cross efficiency	[8]
R.M9:	CSW 1	[13]
R.M10:	CSW 2	[11]
R.M11:	L1-norm	[19]
R.M12:	L1-norm	[19]
R.M13:	Gradient Line	[19]

In the Table 3 ranking order obtained through mentioned methods, in Table 2, are listed. In this table E.D. shows efficient DMUs, and R.Mj, (j = 1, ..., 14) are those explained in the above table. In the following tables E.D stands for efficient DMUs.

Table 3 Ranking orders

E.D.	R.M1	R.M2	R.M3	R.M4	R.M5	R.M6	R.M7	R.M8	R.M9	R.M10	R.M11	R.M12	R.M13
1	7	7	7	6	7	7	7	6	7	7	7	7	7
4	2	2	2	2	2	2	2	2	2	1	2	2	2
7	5	3	4	3	3	4	3	4	1	2	3	5	5
12	6	6	6	5	6	5	5	5	4	3	5	6	6

E.D.	R.M1	R.M2	R.M3	R.M4	R.M5	R.M6	R.M7	R.M8	R.M9	R.M10	R.M11	R.M12	R.M13
15	1	1	1	1	1	1	1	1	3	6	1	1	1
17	3	5	3	5	4	3	4	7	6	5	4	3	3
20	4	4	5	4	5	6	6	3	5	4	6	4	4

As regards of abilities of different ranking models for ranking efficient units and the factors defined by the decision maker consider the following Table 3 and the weights given to each factor.

Now according to the measures obtained through solving different ranking models corresponding weight relate to each method can be obtained. Consider the normalized weights for the mentioned 13 ranking methods as what follows in Table 4.

Table 4 Normalized weights

DMUs	Weights	DMUs	Weights
1	0.070670932	8	0.066634591
2	0.075507771	9	0.084331712
3	0.073628773	10	0.084021309
4	0.086758842	11	0.037954323
5	0.0787944	12	0.044277052
6	0.074246321	13	0.044275951
7	0.073759992		

Now using model (3.3) the results gathered in Table 6. Note that the following results are sorted from minimum to maximum as regard to this fact that the results listed in Table 3 are ranking orders and in voting technique the higher number shows the worse condition in competition thus the results sorted from minimum to maximum.

Table 5 New Ranking order

E.D.	Scale of ranking	Rank order
1	1	7
4	0.387174022	2
7	0.481389565	3
12	0.736710698	4
15	0.301678357	1
17	0.794909307	5
20	0.856721276	6

Comparing the results gathered in Table 5 and results of other ranking models listed in Table 3 it can be seen that most of the units have the same ranking order except unit 12. Thus with hight reliability this method can be used for ranking efficient units. According to the obtained ranking orders it can be concluded that result obtained from the presented model is the aggregation of ranking orders obtained from different DEA models. Consider DMU1 this unit in most of the ranking orders has the worst case as well as the ranking place obtained from the new approach as mentioned in Table 5.

5 Conclusions

In order to provide a new method for ranking efficient units in this paper a method presented which considers some of the important ranking methods, existing in literature. As there does not exist a ranking model considers all the advantages of different ranking models in unified manner, in this paper a method presented with which it is possible to account for advantages of ranking models together. This method is based upon voting technique, performed by DEA method. As regards of the obtained ranking orders form different ranking models and a matrix of weights, corresponds to the different ability of these methods, voting is accounted for in order to consider different aspect of these methods and a new method introduced. For further research on this subject other aspect of other subjects in MCDM technique can also be accounted for in order to obtain a new ranking order on basis of the existing ranking methods.

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