

Multi-objective modeling for post-disaster water distribution network reconstruction under interval uncertainty

M. Farnam*

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Abstract Urban water distribution networks are vital infrastructures, yet natural disasters such as earthquakes and floods cause severe damage to them. Reconstruction presents a strategic opportunity to enhance resilience, though it inherently involves conflicting objectives of maximizing resilience and distributive equity while minimizing cost and pollution. Uncertainty in input data including cost, project impacts, and pollution due to market fluctuations, climate changes, and measurement errors further complicates decision-making. This study develops a multi-objective 0-1 knapsack optimization model with interval data for post-disaster water supply network reconstruction project portfolio selection. The model incorporates four interval-valued objective functions for maximizing resilience and equity and minimizing cost and pollution, subject to constraints on budget, pollution, parallel execution time, minimum reliability, and regional project quotas. Interval solutions are compared and ranked using the possibility degree of superiority and a Boolean matrix, avoiding any early defuzzification. Objectives are normalized by their maximum possible values, and complete enumeration is employed for a 12-project numerical study. Five weighting scenarios are defined, accompanied by sensitivity analysis on four critical parameters. Results indicate that the resilience-oriented scenario achieves the highest resilience and equity, though at higher cost and pollution, whereas the economy-oriented scenario minimizes cost and the environment-oriented scenario yields the lowest pollution. Sensitivity analysis identifies budget as the most critical constraint; under severe budget or carbon caps, all scenarios converge to a single solution. The resilience-oriented scenario proves stable against tighter reliability but fragile against budget cuts, while the economy-oriented scenario exhibits opposite behavior. The proposed model provides a robust decision-support tool, revealing that under extreme resource scarcity, policymakers must prioritize resource augmentation. Future research may explore project synergies and chance constraints with variable confidence levels.

Keywords: Multi-objective Modeling, Water Distribution Network Reconstruction, Interval Uncertainty, Possibility Degree.

1 Introduction

Urban water distribution networks, as the most vital infrastructures of any society, play an undeniable role in maintaining public health, continuity of economic activities, and social sustainability. Experiences from recent decades have shown that natural events such as

* Corresponding Author. (✉)

E-mail: m.farnam@scu.ac.ir (M. Farnam)

M. Farnam

Department of Electrical Engineering, Shohadaye Hoveizeh Campus of Technology, Shahid Chamran University of Ahvaz, Dasht-e Azadegan, Khuzestan, Iran.

earthquakes, severe floods, and prolonged droughts can cause heavy damage to these networks and lead to secondary consequences including the spread of waterborne diseases, forced migration, and industrial shutdowns [1, 2]. In such conditions, reconstruction of water supply networks is not merely about restoring the pre-disaster state, but rather a valuable opportunity to enhance system resilience against future events [1, 3, 4]. The problem of selecting a reconstruction project portfolio is inherently multi-objective, involving conflicting objectives and various constraints. On one hand, policymakers seek to maximize resilience (the system's ability to absorb shocks and recover quickly) and distributive equity (balanced allocation of resources among different regions) [2, 4, 5]. On the other hand, severe financial and time constraints (which play a critical role in post-disaster reconstruction) together with environmental requirements (reduction of pollution from construction activities) force them to minimize costs and pollution [6, 7]. In addition, technical constraints such as minimum reliability and regional project quotas further complicate the decision space [1, 8].

A fundamental challenge overlooked in many practical models is uncertainty in input data. In the real world, exact values of execution cost, the actual impact of a project on resilience, the amount of pollution, and even equity indicators cannot be determined with certainty. Sources of this uncertainty include: fluctuations in construction material prices in the post-disaster market, climate changes affecting pollution patterns, inherent errors in field measurements, and disagreements among experts and stakeholders [9, 10]. Common approaches for modeling uncertainty include probabilistic methods, fuzzy theory, and interval numbers. In post-disaster conditions where time and information are limited, interval numbers, requiring only the lower and upper bounds of each parameter, are a practical, transparent, and computationally efficient option [10, 11, 12]. As Liao et al. [12] showed in their comprehensive review, interval techniques have received increasing attention in multi-criteria decision-making because they prevent information loss due to defuzzification.

In recent years, numerous studies have addressed infrastructure reconstruction optimization with various approaches. Some studies focused on reducing time and cost using methods such as maximum flow theory [13]; others pursued multi-objective approaches like time-cost-quality trade-off analysis [7]; and some incorporated environmental considerations and water resource sustainability (including the concept of water neutrality) into their models [6, 14]. However, a systematic literature review reveals that no existing study has simultaneously addressed all these aspects under interval uncertainty with a possibility-degree-based ranking approach. Most existing studies either ignore uncertainty or defuzzified it by using the upper bound or the average of the interval, which leads to information loss and sometimes unrealistic solutions [15, 16].

To address these limitations, the present research offers a multi-objective 0-1 knapsack optimization model with interval coefficients. The main innovations are:

1. Full preservation of interval information in all objective functions and constraints, without any early defuzzification, and presenting final objective values as intervals.
2. Using the concept of possibility degree to compare and rank interval solutions [16, 17, 18, 19, 20], instead of simple methods such as comparing mid-points or upper bounds that discard information.
3. Combining four objectives (maximizing resilience and equity, minimizing cost and pollution) with realistic constraints including budget, carbon cap, parallel execution time, minimum reliability, and regional quota.
4. Comprehensive sensitivity analysis on four key parameters (minimum reliability, maximum total completion time, budget, and carbon cap) to evaluate solution stability under five different weighting scenarios (resilience-oriented, economy-oriented, equity-oriented, environment-oriented, and balanced).

The structure of the paper is as follows: After this introduction, Section 2 reviews the literature. Section 3 describes the mathematical model and concepts of interval theory. Section 4 presents the solution method based on normalization and ranking with the possibility degree. Section 5 reports the numerical study results and sensitivity analysis. Section 6 and 7 provides some managerial insights and conclusions respectively.

2 Literature Review

In this section, we review findings from previous studies in four areas related to the problem.

2.1 Multi-objective models in critical infrastructure reconstruction

Research on post-disaster infrastructure network reconstruction began in the 1990s with an emphasis on minimizing cost and time [7, 13]. Over time, researchers realized that qualitative objectives such as resilience and equity also play a decisive role in long-term reconstruction success. Bruneau et al. [4] introduced a quantitative framework for seismic resilience, which later became the basis for many optimization models. Zhang et al. [1] proposed a three-stage method for assessing seismic resilience of water distribution networks based on user demand and performed multi-objective optimization using NSGA-II. Also, Miao et al. [2] presented a four-stage process-based framework for seismic resilience assessment that explicitly includes pre-disaster preparedness, resistance, recovery, and long-term adaptation stages. In the water networks field, Beygi et al. [8] developed a two-objective optimization model for design and operation of water transmission systems using the Power Resilience Index as a measure of hydraulic reliability. However, most of these models cover only two or three objectives and rarely consider four conflicting objectives simultaneously. In particular, combining resilience, cost, distributive equity, and pollution in a single framework is seldom seen [2, 5, 6].

2.2 Optimization with interval data and ranking methods

Uncertainty in input data (e.g., cost, project impact on resilience, and pollution level) is a serious challenge for realistic modeling. The interval approach, due to its simplicity and lack of need for precise statistical information, has gained attention in the last two decades [10, 12]. Ishibuchi and Tanaka [21] performed one of the first works on linear optimization with interval coefficients. In the knapsack problem context, Kasperski and Kulej [22] investigated the knapsack problem with fuzzy costs and proposed a robust approach based on possibility theory. Dalouchei et al. [15] presented a two-objective model for scheduling construction projects using the critical chain method and triangular interval fuzzy numbers. However, these methods often rely on some form of defuzzification (e.g., converting to crisp numbers).

An important advance is the introduction of the possibility degree of superiority for comparing uncertain quantities. Xu and Da [20] proposed one of the first analytical relations for calculating the probability that one interval dominates another. This concept was later extended to more complex structures such as hesitant interval-valued fuzzy sets [17], where a fuzzy membership component is attached to the interval bounds. Notably, when the fuzzy membership component is omitted and only the interval bounds are retained, the possibility degree formula reduces directly to a pure interval comparison. Liu et al. [16] proved the equivalence of two common

possibility-degree formulas and proposed a generalized version that incorporates the decision-maker's attitude. Deqing et al. [19] provided a comprehensive review of interval number ranking methods and confirmed that the method based on the possibility degree matrix with a modified Boolean matrix preserves order stability. In this study, we adopt this reduced, purely interval-based form of the possibility degree, as it aligns naturally with our model's interval data structure and avoids any early defuzzification. Nevertheless, the application of these interval ranking methods to multi-objective knapsack problems with four objectives and regional quota constraints is still limited. Many existing studies still use simple defuzzification (e.g., replacing an interval with its mean or upper bound), which discards important information [11, 15, 22].

2.3 Distributive equity and environmental considerations in project selection

In recent years, two additional aspects have been added to project selection models: Distributive equity (fair allocation of resources among regions or population groups) and environmental sustainability (reduction of carbon and other pollution). Marsh and Schilling [18] were among the first to add distributive equity constraints to location models. In disaster management, Kamyabniya et al. [23], in their comprehensive review of optimization models for disaster response operations, showed that distributive equity in relief item distribution and shelter location has received increasing attention, but significant gaps remain. In the area of carbon pollution, He et al. [6] presented a multi-objective optimization model for regional energy generation mix considering cap-and-trade mechanisms and renewable energy portfolio standards. Also, Mwafy et al. [14] addressed the concept of water neutrality and the role of effective water management in achieving sustainability. Furthermore, fair financing of water projects and attention to policy aspects of resource distribution have recently gained attention [24]. However, simultaneously combining distributive equity and environment with the traditional objectives of cost and resilience in a 0-1 knapsack model with interval data has rarely been done. In particular, introducing a regional quota (minimum number of projects in each region) as an explicit constraint to ensure equity is not seen in most studies.

2.4 Solution methods for multi-objective knapsack problems

The multi-objective 0-1 knapsack problem, due to its NP-hard nature, requires efficient solution methods [5, 25]. For small dimensions (typically $n \leq 20$), exhaustive enumeration can guarantee the global optimal solution and serves as a benchmark for validating approximate methods [25]. For larger dimensions, multi-objective metaheuristic algorithms such as NSGA-II [26], MOEA/D [27], and SPEA2 [28] are widely used. Scalarization methods such as weighted sum and epsilon-constraint are also commonly employed to convert a multi-objective problem into a single-objective one [29]. In more recent research, Nowak and Trzaskalik [30] proposed an interactive trade-off analysis procedure within a multi-objective stochastic dynamic programming framework with a finite horizon. Also, Xiao et al. [31] presented a hybrid algorithm with a dual-search pattern based on Gaussian distribution and an extended version of the Firefly Algorithm for financial resource allocation under budget constraints. Wang et al. [32] also designed an extended firefly algorithm aiming to balance convergence and diversity in search [32]. In the field of fuzzy modeling, Singh [33] investigated the fuzzy knapsack problem with single-objective and two-objective functions using triangular fuzzy numbers, and

Niksirat and Nasserli [34] transformed fuzzy models into equivalent crisp linear models using a credibility-based ranking method.

In the present study, due to the 12 projects, complete enumeration is used to obtain non-dominated interval solutions for each weighting scenario. Weights in the weighted sum method are selected based on five different scenarios (Table 3), and sensitivity analysis with respect to weights is performed for validation.

2.5 Research gap and innovation of the present study

Table 1 provides a systematic comparison of the proposed model with representative recent studies across five key dimensions: number of objectives, uncertainty type, ranking without defuzzification, solution approach, and application domain.

Table 1 Comparison of the proposed model with representative recent studies.

Study	Multi-Objective Programming	Uncertainty Type	Ranking without Defuzzification	Solution Approach	Application
Zhang et al. [1]	Yes (2 objectives)	—	—	NSGA-II	Water network
Miao et al. [2]	Yes (2 objectives)	—	—	Framework-based assessment	Water network
He et al. [6]	Yes (3 objectives)	—	—	Scenario-based optimization	Power generation
Dalouchei et al. [15]	Yes (2 objectives)	Fuzzy (triangular)	No (mid-point)	Goal programming	Project scheduling
Zeng et al. [17]	No (Theoretical)	Interval	Yes (Possibility degree)	Theoretical ranking	Interval comparison
Kasperski and Kulej [22]	Yes (2 objectives)	Fuzzy	No (possibility-based robust)	Robust optimization	Knapsack problem
This study	Yes (4 objectives)	Interval	Yes (Possibility degree)	Integrated framework: Normalization, Weighted sum and Possibility-degree ranking	Water network

As the Table 1 clearly shows, no previous study has simultaneously combined all the features that distinguish our work: (1) four conflicting objectives in a single model, (2) full preservation of interval information without any defuzzification, (3) ranking of interval solutions using the possibility degree of superiority with a Boolean matrix, and (4) application to post-disaster water distribution network reconstruction under combined constraints including budget, carbon cap, parallel execution time, minimum reliability, and regional quota.

In response to these identified gaps, the present research offers a multi-objective 0-1 knapsack optimization model with interval data that simultaneously combines four conflicting objectives with realistic constraints. The main innovations are: (1) full preservation of interval information at all modeling and solution stages, without any early defuzzification; (2) use of the possibility degree of superiority [16, 17, 19, 20] to compare and rank interval solutions; (3) design of a solution process including normalization based on maximum possible values, weighted scalarization with five different scenarios, and final ranking using the possibility degree matrix and the Boolean matrix [17, 19]; (4) systematic sensitivity analysis on four key parameters showing the stability of each weighting scenario. These features, which are uniquely

combined in this study, distinguish the proposed model from previous studies and make it suitable for application in real, highly uncertain post-disaster conditions.

3 Modeling and solution process

In this section, the mathematical model for the post-disaster water supply network reconstruction project portfolio selection problem is presented. Selecting an optimal infrastructure project portfolio in post-disaster conditions is a strategic decision with long-term consequences. Due to their vital nature, water supply networks are exposed to various natural threats such as earthquakes, floods, droughts, and others. Reconstructing these networks requires not only restoring the pre-disaster state but also increasing resilience against future events. On the other hand, financial, temporal, and environmental resources are limited, and the fair distribution of projects among different regions is also an ethical and social requirement. Therefore, the problem is inherently multi-objective and includes conflicting objectives such as maximizing resilience and equity against minimizing cost and pollution.

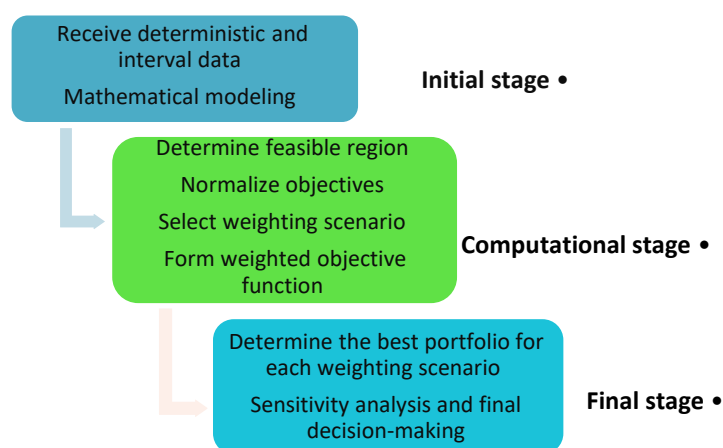


Fig. 1 Overall structure and solution process.

In addition to the complexity arising from multiple objectives, uncertainty in data is a fundamental challenge. In practice, exact values of cost, environmental impacts, and even resilience and equity indicators cannot be determined with certainty. Several reasons for this uncertainty exist, including disagreements among experts and decision-makers, market and material price fluctuations, climate changes affecting pollution, and measurement errors. Modeling this uncertainty with interval numbers is a common and practical approach that, instead of assuming a probability distribution (which requires extensive experimental data) or fuzzy numbers (which require determining membership functions), deals only with the lower and upper bounds of each value. This approach is computationally efficient and transparent in interpreting results.

3.1 Mathematical modeling

The mathematical structure of the problem is of the multi-objective 0-1 knapsack type because each project is either selected completely or not selected. Constraints include resource limits

such as budget, time, pollution, and technical requirements including minimum reliability and regional quota.

Table 2 List of symbols, parameters, and decision variables.

Category	Symbol	Description
Indices and sets	j	Index of candidate projects, $j = 1, 2, \dots, n$
	k	Index of regions, $k = 1, 2, \dots, k$
	$\mathcal{P}$	Set of candidate projects
	$\mathcal{D}_k$	Set of projects executable in region k
Deterministic parameters	t_j	Execution time of project j
	r_j	Reliability gains of project j
	b^{max}	Maximum available budget
	e_{cap}^{max}	Maximum allowable pollution
	t^{max}	Maximum allowable execution time per project
	r^{min}	Minimum required total reliability
	δ_k	Minimum number of projects allocated to region k .
Interval parameters	$[a_j^l, a_j^u]$	Interval resilience gain from project j
	$[c_j^l, c_j^u]$	Interval cost of project j
	$[\alpha_j^l, \alpha_j^u]$	Interval distributive equity gain from project j
	$[e_j^l, e_j^u]$	Interval pollution from project j
Decision variables	x_j	Binary decision variable (1 if project j is selected, 0 otherwise)

The model is presented in a logical step-by-step sequence: first, the indices and sets are defined; second, the parameters (both deterministic and interval) are introduced; third, the decision variables are described; fourth, the objective functions are formulated; and finally, the constraints (both deterministic and interval) are specified.

To enhance clarity and facilitate reproducibility, all mathematical symbols, indices, parameters, and decision variables employed in the model are systematically defined in Table 2.

3.1.1 Indices and sets

$\mathcal{P}$: Set of candidate projects ($\mathcal{P} = \{1, 2, \dots, n\}$, where n is the number of projects).

k : Set of regions in the considered area ($k = \{1, 2, \dots, k\}$, where k is the number of regions).

$\mathcal{D}_k$: Set of candidate projects to be executed in region k .

The set $\mathcal{D}_k$ for each region contains specific and distinct projects.

3.1.2 Parameters

Two categories of parameters, deterministic and interval, are introduced as follows.

i. Deterministic parameters

These parameters have specific and unambiguous values and are extracted from technical data or policy approvals.

t_j : Execution time of project j .

r_j : Amount of network reliability gained by executing project j .

b^{max} : Maximum available budget resources.

e_{cap}^{max} : Maximum amount of pollution set by the policymaker for all selected projects .

t^{max} : Maximum execution time per project.

r^{min} : Minimum expected reliability for all selected projects. This value represents the policymaker's risk acceptance level.

δ_k : Minimum number of projects allocated to region k .

ii. Interval parameters

These parameters are presented as intervals, but with specified lower and upper bounds.

$[a_j^l, a_j^u]$: Interval increase in resilience from executing project j .

$[c_j^l, c_j^u]$: Interval increase in cost from executing project j .

$[\alpha_j^l, \alpha_j^u]$: Interval increase in distributive equity from executing project j .

$[e_j^l, e_j^u]$: Interval increase in pollution from executing project j .

3.1.3 Decision variables

x_j : Binary decision variable ($x_j \in \{0,1\}$), where $x_j = 1$ means project j is selected and $x_j = 0$ means project j is not selected.

3.1.4 Objective functions

The model includes four objective functions, all defined as intervals and with different optimization directions (maximization or minimization).

$$\bar{Z}_1 = \max \sum_j [a_j^l, a_j^u] x_j \quad (1)$$

$$\bar{Z}_2 = \min \sum_j [c_j^l, c_j^u] x_j \quad (2)$$

$$\bar{Z}_3 = \max \sum_j [\alpha_j^l, \alpha_j^u] x_j \quad (3)$$

$$\bar{Z}_4 = \min \sum_j [e_j^l, e_j^u] x_j \quad (4)$$

Equation (1) is the first objective, aiming to maximize the total increase in resilience from the selected projects. Through this equation, we seek to achieve the maximum ability of the system to withstand various crises and return to normal conditions. Equation (2) is the second objective, set to minimize the total cost of the selected projects. Equation (3), the third objective, aims to maximize the total equity indicators from the selected projects. Here, equity means fair distribution of resources among different regional divisions. Equation (4), in accordance with environmental requirements and organizational commitments, limits the amount of pollution. The purpose of this equation is to select projects with lower pollution.

3.1.5 Constraints

The constraints are categorized into deterministic and interval types as presented below.

i. Deterministic constraints

$$t_j x_j \leq t^{max}, \forall j \quad (5)$$

$$\sum_{j \in \mathcal{D}_k} x_j \geq \delta_k, \forall k \quad (6)$$

$$\sum_j r_j x_j \geq r^{min} \quad (7)$$

Constraint (5) – Time limit: This constraint imposes an upper bound on the execution time of each individual project. In practice, reconstruction projects cannot exceed a certain duration due to resource availability, workforce constraints, and the urgent need to restore water services to affected communities. This constraint ensures that no single project becomes a bottleneck in the overall reconstruction timeline. By limiting the maximum allowable time per project, this constraint directly shapes the feasible region by excluding any project with an execution time longer than t^{max} . It interacts with the parallel execution assumption, meaning that the total reconstruction time is determined by the maximum duration among selected projects, rather than their sum.

Constraint (6) – Regional quota: This constraint enforces distributive equity by ensuring that each region receives a minimum number of reconstruction projects. In real-world post-disaster scenarios, reconstruction resources are often concentrated in economically or politically prioritized regions, leaving underserved areas with inadequate support. This constraint explicitly prevents such concentration by mandating that region k must have at least δ_k projects in the final portfolio. The value of δ_k can be set proportionally to the population, damage severity, or relative deprivation of each region. Without this constraint, the model tends to select projects solely based on cost or resilience efficiency, which may exacerbate regional disparities. The presence of this constraint thus shapes the feasible region by excluding any portfolio that does not meet the regional minimums, and it interacts with the budget constraint by potentially forcing the selection of higher-cost projects in less efficient regions.

Constraint (7) – Minimum reliability: This constraint guarantees that the overall technical reliability of the reconstructed network does not fall below a minimum acceptable threshold. This reflects the policymaker's risk acceptance level and ensures that the network can maintain adequate performance after reconstruction. In practice, reliability is a measure of the network's ability to deliver water under various demand and failure scenarios. By requiring that the sum of reliability gains from selected projects exceeds r^{min} , this constraint excludes portfolios that, while cost-efficient or environmentally friendly, would leave the network in a technically vulnerable state. This constraint interacts with the budget and carbon caps, as projects with higher reliability gains often come with higher costs and pollution levels.

$$\sum_j [c_j^l, c_j^u] x_j \leq b^{max} \quad (8)$$

$$\sum_j [e_j^l, e_j^u] x_j \leq e_{cap}^{max} \quad (9)$$

ii. Interval constraints

Constraint (8) – Budget constraint: This constraint ensures financial feasibility by limiting the total cost of selected projects to the available budget. Since project costs are uncertain and expressed as intervals, a pessimistic (risk-averse) approach is adopted: the upper bound of the interval cost sum must not exceed the budget. This guarantees that even under the worst-case cost scenario, the selected portfolio remains within financial constraints. This approach is particularly suitable for post-disaster reconstruction, where budget overruns can have severe consequences. Without this constraint, the model could select portfolios that are financially infeasible. The budget constraint is the most critical constraint in the model, as it directly limits the number and type of projects that can be selected.

Constraint (9) – Carbon cap: This constraint limits the total pollutions from reconstruction activities to a maximum allowable level, reflecting regulatory commitments and environmental sustainability goals. Similar to the budget constraint, a pessimistic approach is adopted: the

upper bound of the interval pollution sum must not exceed the carbon cap. This ensures that even under the worst-case emission scenario, the environmental impact remains within acceptable limits. This constraint is particularly relevant in regions with strict environmental regulations or international climate commitments. The carbon cap interacts with the budget and reliability constraints, as projects that are more resilient or equitable often have higher pollution levels, requiring careful trade-off analysis.

iii. Constraint (10) – Binary condition:

$$x_j \in \{0,1\}, \quad \forall j, \quad (10)$$

The binary condition in Constraint (10) ensures that each project is either fully selected or not selected, reflecting the fundamental characteristic of the 0-1 knapsack problem. This means that partial implementation of projects is not allowed, which aligns with real-world reconstruction practice where projects are typically approved and funded as indivisible units. This binary nature significantly impacts the feasible region by restricting the set of possible portfolios to discrete combinations, making the problem NP-hard.

3.1.6 Feasible region

Based on the above constraints, the feasible region F of the model is formally defined as:

$$F = \{x \in \{0,1\}^n \mid \text{constraints (5) through (10) are satisfied}\} \quad (11)$$

This region contains all project portfolios that simultaneously meet the time, regional quota, reliability, budget, carbon, and binary restrictions. Any solution outside this region is infeasible and is not considered in the optimization process.

Each constraint shapes the feasible region as follows:

- The time limit (5) excludes projects longer than t^{max} .
- The regional quota (6) removes portfolios that fail to meet the minimum number of projects in any region.
- The reliability requirement (7) eliminates portfolios with insufficient technical performance.
- The budget cap (8) and the carbon limit (9) adopt a pessimistic (upper-bound) approach to ensure feasibility under worst-case cost and emission scenarios.
- The binary condition (10) restricts the search space to discrete combinations.

3.1.7 Model summary

The overall model, referred to as Model 1, can be summarized as follows:

$$\bar{Z}_1 = \max \sum_j [a_j^l, a_j^u] x_j \quad (1)$$

$$\bar{Z}_2 = \min \sum_j [c_j^l, c_j^u] x_j \quad (2)$$

$$\bar{Z}_3 = \max \sum_j [\alpha_j^l, \alpha_j^u] x_j \quad (3)$$

$$\bar{Z}_4 = \min \sum_j [e_j^l, e_j^u] x_j \quad (4)$$

Subject to

$$t_j x_j \leq t^{max}, \forall j \quad (5)$$

$$\sum_{j \in \mathcal{D}_k} x_j \geq \delta_k, \forall k \quad (6)$$

$$\sum_j r_j x_j \geq r^{\min} \quad (7)$$

$$\sum_j [c_j^l, c_j^u] x_j \leq b^{\max} \quad (8)$$

$$\sum_j [e_j^l, e_j^u] x_j \leq e_{cap}^{\max} \quad (9)$$

$$x_j \in \{0,1\}, \quad \forall j \quad (10)$$

Overall, the proposed model is a complete and realistic model for selecting reconstruction projects that simultaneously considers the fourfold objectives: economic, resilience-and-equity-oriented, and environmental. Moreover, being constrained by diverse constraints can correctly reflect the complexity of the real world.

3.2 Interval theory and theoretical analysis

Using the possibility degree, intervals can be compared directly without defuzzification, which often leads to information loss. This idea uses the concept of the degree of superiority of one interval over another. For two intervals $A_1 = [a_1^l, a_1^u]$ and $A_2 = [a_2^l, a_2^u]$, the possibility degree of A_1 over A_2 is defined as:

$$p(A_1 \geq A_2) = \max \left\{ 1 - \max \left\{ \frac{a_2^u - a_1^l}{\text{len}(A_1) + \text{len}(A_2)}, 0 \right\}, 0 \right\} \quad (12)$$

where $\text{len}(A_1) = a_1^u - a_1^l$ and $\text{len}(A_2) = a_2^u - a_2^l$.

Using the concept of possibility degree, we can conclude:

$$A_i \geq A_j \Leftrightarrow p(A_i \geq A_j) \geq 0.5 \quad (13)$$

Using equation (11) and (12) in the above definition, the following theorem holds.

Theorem 1 [14]. Suppose we have three interval numbers $A_i = [a_i^l, a_i^u]$, $A_j = [a_j^l, a_j^u]$, and $A_k = [a_k^l, a_k^u]$. Then the following relations hold:

- 1) $0 \leq p(A_i \geq A_j) \leq 1$,
- 2) $p(A_i \geq A_j) + p(A_j \geq A_i) = 1, p(A_i \geq A_i) = 0.5$,
- 3) $p(A_i \geq A_j) = 1 \Leftrightarrow a_i^l \geq a_j^u$,
- 4) $p(A_i \geq A_j) = 0 \Leftrightarrow a_i^u \leq a_j^l$,
- 5) $p(\bar{a}_1 \geq \bar{a}_2) \geq 0.5 \Leftrightarrow a_i^l + a_i^u \geq a_j^l + a_j^u$,
- 6) $(p(A_i \geq A_j) \geq 0.5, p(A_j \geq A_k) \geq 0.5) \Rightarrow p(A_i \geq A_k) \geq 0.5$.

Next, a method based on the possibility degree for ranking a set of interval numbers is presented. In this method, for ranking m interval data, we form a possibility matrix $P = (p_{ij})_{m \times m}$. In this matrix, p_{ij} represents the possibility that interval i dominates interval j . Each A_i is compared with all A_j ($j = 1, 2, \dots, n$) using equations (11) and (12).

The Boolean matrix $Q = (q_{ij})_{m \times m}$ is derived from P as follows:

$$q_{ij} = \begin{cases} 1, & p_{ij} \geq 0.5 \\ 0, & p_{ij} < 0.5 \end{cases} \quad (14)$$

Matrix Q is the ranking matrix for the m intervals $A_1, A_2, \dots, A_m$.

For each A_i ($i = 1, 2, \dots, m$), let $\theta_i = \sum_{j=1}^m q_{ij}$; then the rank vector $\theta = (\theta_1, \theta_2, \dots, \theta_m)$ is obtained. In this vector, the θ_i values are sorted in descending order.

Theorem 2 [1^V]. Suppose we have m interval numbers $A_1, A_2, \dots, A_m$. Furthermore, suppose $P = (p_{ij})_{m \times m}$ and $Q = (q_{ij})_{m \times m}$ are the corresponding possibility degree and ranking matrices for the given intervals, respectively. Then if $\theta_i = \sum_{j=1}^m q_{ij}$, we have:

$$\theta_i \geq \theta_j \Leftrightarrow p_{ij} \geq 0.5 \quad (15)$$

Theorem 2 shows that the stated conditions are sufficient for ranking a set of intervals.

Based on the above material, Algorithm 1, summarizes the steps of process for ranking m interval elements.

Algorithm 1. Process for ranking m interval numbers based on the possibility matrix.

Input: m interval numbers $A_1, A_2, \dots, A_m$
 Step 1: Write the possibility degree matrix $P = (p_{ij})_{m \times m}$ after finding the possibility degree for each pair of elements.
 Step 2: Construct the Boolean matrix $Q = (q_{ij})_{m \times m}$.
 Step 3: Form the rank vector $\theta = (\theta_1, \theta_2, \dots, \theta_m)$ by calculating the row sums of the matrix .
 Step 4: Sort the values θ_i ($i = 1, 2, \dots, m$) in descending order.
 Step 5: Rank the intervals according to the θ_i values. The interval corresponding to the larger θ_i has a higher rank.
 Output: Sorted m interval numbers.

Definition 1. Model 1 with decision space F is called feasible if there exists $x \in F$ as a set of projects that satisfies constraints (5) to (10).

Theorem 3. Model 1 is NP-hard in general.

Proof. Since the problem belongs to the class of 0-1 knapsack problems, the proof is straightforward.

3.3 Solution method

In this section, the algorithmic steps for solving Model 1 are described. Given the small size of the problem in the numerical study, the complete enumeration method is used. This method, while guaranteeing the global optimal solution for each weighting scenario, provides a basis for validating approximate methods for larger dimensions. The proposed algorithm consists of five main steps, detailed below.

Step 1: Generation of the feasible space

Among the set of candidate projects, all combinations $x_j \in \{0, 1\}^{12}$ that satisfy the constraints are considered as the set of feasible solutions. Let F be the set of all feasible solutions of the problem.

Step 2: Normalization of objectives using the maximum possible value

The four objectives have different units and ranges. To combine them into a scalar objective function, normalization to the interval $[0, 1]$ is applied. This method is independent of the solution process and operates based on the maximum possible value of each objective. For this purpose, compute the following values:

$$M_1 = \sum_j a_j^u, M_2 = \sum_j c_j^u, M_3 = \sum_j \alpha_j^u, M_4 = \sum_j e_j^u \quad (16)$$

Then for maximization objectives we have:

$$\bar{Z}_1 = \max \sum_j \left[\frac{a_j^l}{M_1}, \frac{a_j^u}{M_1} \right] x_j, \bar{Z}_3 = \max \sum_j \left[\frac{\alpha_j^l}{M_3}, \frac{\alpha_j^u}{M_3} \right] x_j \quad (17)$$

And for minimization objectives, converting to maximization via complement:

$$\bar{\bar{Z}}_2 = \max \sum_j \left[1 - \frac{c_j^u}{M_2}, 1 - \frac{c_j^l}{M_2} \right] x_j, \bar{\bar{Z}}_4 = \max \sum_j \left[1 - \frac{e_j^u}{M_4}, 1 - \frac{e_j^l}{M_4} \right] x_j \quad (18)$$

Step 3: Weighted scalarization and formation of the combined objective function

Based on the decision-maker's priorities, the objective weights are selected from among the five weighting policies presented in Table 3. For each feasible solution x and each weighting scenario, the combined interval objective function is defined as the weighted sum of the normalized intervals:

$$\bar{\bar{Z}}(x) = \sum_{g=1}^4 w_g \bar{\bar{Z}}_g(x) = [\bar{\bar{Z}}^l(x), \bar{\bar{Z}}^u(x)] = \left[\sum_{g=1}^4 w_g \bar{\bar{Z}}_g^l(x), \sum_{g=1}^4 w_g \bar{\bar{Z}}_g^u(x) \right] \quad (19)$$

This interval represents the total utility spectrum for feasible solution x under the given weights.

Table 3 Weighting scenarios based on objective weight.

Scenario	Description	objective weights			
		w_1	w_2	w_3	w_4
S_1	Resilience-oriented	0.70	0.1	0.1	0.1
S_2	Economy-oriented	0.1	0.70	0.1	0.1
S_3	Equity-oriented	0.1	0.1	0.70	0.1
S_4	Environment-oriented	0.1	0.1	0.1	0.70
S_5	Balanced	0.25	0.25	0.25	0.25

Theorem 4. (Properties of normalization).

For every feasible solution x , the normalized combined objective function satisfies:

$$0 \leq \bar{\bar{Z}}^l(x) \leq \bar{\bar{Z}}^u(x) \leq 1 \quad (20)$$

Furthermore, the normalization is monotone with respect to the interval order: for any two feasible solutions x and y , if $\bar{\bar{Z}}_g(x) \geq \bar{\bar{Z}}_g(y)$ for all g , then $\bar{\bar{Z}}(x) \geq \bar{\bar{Z}}(y)$.

Proof. The first part follows directly from the fact that each normalized objective $\bar{\bar{Z}}_g(x)$ lies in $[0,1]$ (by construction) and the weights are non-negative and sum to one. The monotonicity property is a direct consequence of the linearity of the weighted sum and the preservation of interval order under the normalization mapping, which is strictly increasing and linear for each objective type.

Step 4: Ranking intervals using the possibility degree matrix

After computing $\bar{\bar{Z}}(x)$ for all $x \in F$, a set of intervals is obtained. To select the best solution for each scenario, the concept of the possibility degree (Algorithm 1) is used.

Step 5: Extracting results based on the weighting scenario and analyzing results

Go back to Step 3 and select another weighting scenario. After determining the best solution for each scenario, analyze the results and select the appropriate option.

Remark 1 (Satisfaction of interval constraints).

In constraints (8) and (9), the left-hand side is an interval number. Since the decision-maker must be confident that even under the worst-case scenario the actual cost does not exceed the approved budget, the condition that the sum of the upper bounds of the interval costs stays within the budget is adopted as the feasibility criterion. The same applies to the carbon emission constraint. This approach does not imply defuzzifying or scoring the objective functions; the objectives remain interval-valued and are ranked using the possibility degree. Distinguishing between “reliability-type constraints” (which need to be guaranteed) and “objectives” (whose interval flexibility is preserved) is one of the strengths of the proposed model.

4 Numerical study and findings

4.1 Numerical study setup

In this section, we present a numerical study designed to validate the proposed model and demonstrate its practical applicability. The parameter values were systematically generated to reflect realistic ranges for costs, resilience, equity, and pollution, and were calibrated through multiple iterations to ensure feasibility across all constraints. This approach is standard in optimization studies for demonstrating model performance when access to complete and reliable post-disaster data is inherently limited. The structure of the data was derived directly from the proposed multi-objective model itself to ensure consistency between the model and the numerical example. We aim to select an optimal investment portfolio from 12 projects for reconstructing the water supply network infrastructure. For this purpose, the model parameters based on the decision-makers' opinions are presented in Table 4.

The other parameters of the problem are:

$$b^{max} = 380, e_{cap}^{max} = 1700, t^{max} = 30, r^{min} = 1.2, \delta_1 = 1, \delta_2 = 1, \delta_3 = 2 \\ \mathcal{D}_1 = \{2, 3, 7, 8\}, \mathcal{D}_2 = \{1, 6, 11, 12\}, \mathcal{D}_3 = \{4, 5, 9, 10\}.$$

Table 4 Basic information for reconstruction of water supply network projects.

Project code	Region	$[a_j^l, a_j^u]$	$[c_j^l, c_j^u]$	$[\alpha_j^l, \alpha_j^u]$	$[e_j^l, e_j^u]$	t_j	r_j
P01	Center	[0.25,0.55]	[64,72]	[0.20,0.40]	[150,240]	22	0.20
P02	North	[0.20,0.50]	[40,50]	[0.15,0.35]	[140,250]	20	0.28
P03	North	[0.15,0.35]	[15,24]	[0.65,0.85]	[70,130]	12	0.12
P04	South	[0.20,0.45]	[38,56]	[0.75,0.95]	[150,260]	24	0.22
P05	South	[0.15,0.40]	[28,42]	[0.70,0.90]	[95,185]	18	0.18
P06	Center	[0.10,0.30]	[32,48]	[0.25,0.45]	[110,210]	20	0.15
P07	North	[0.25,0.55]	[55,82]	[0.30,0.50]	[150,390]	30	0.30
P08	North	[0.08,0.20]	[7,12]	[0.60,0.80]	[30,70]	6	0.08
P09	South	[0.18,0.42]	[35,52]	[0.70,0.90]	[130,200]	22	0.20
P10	South	[0.15,0.38]	[29,44]	[0.45,0.65]	[90,110]	20	0.18
P11	Center	[0.05,0.18]	[11,19]	[0.35,0.55]	[120,250]	10	0.30
P12	Center	[0.20,0.45]	[45,65]	[0.20,0.40]	[160,290]	22	0.32

Based on Model 1 and the given parameters, the mathematical modeling of the problem is summarized as follows.

$$\begin{aligned} \max Z_1 &= [0.25,0.55]x_1 + [0.20,0.50]x_2 + [0.15,0.35]x_3 + [0.20,0.45]x_4 \\ &\quad + [0.15,0.40]x_5 + [0.10,0.30]x_6 + [0.25,0.55]x_7 + [0.08,0.20]x_8 \\ &\quad + [0.18,0.42]x_9 + [0.15,0.38]x_{10} + [0.05,0.18]x_{11} + [0.20,0.45]x_{12} \\ \min Z_2 &= [64,72]x_1 + [40,50]x_2 + [15,24]x_3 + [38,56]x_4 + [28,42]x_5 + [32,48]x_6 \\ &\quad + [55,82]x_7 + [7,12]x_8 + [35,52]x_9 + [29,44]x_{10} + [11,19]x_{11} \\ &\quad + [45,65]x_{12} \\ \max Z_3 &= [0.20,0.40]x_1 + [0.15,0.35]x_2 + [0.65,0.85]x_3 + [0.75,0.95]x_4 \\ &\quad + [0.70,0.90]x_5 + [0.25,0.45]x_6 + [0.30,0.50]x_7 + [0.60,0.80]x_8 \\ &\quad + [0.70,0.90]x_9 + [0.45,0.65]x_{10} + [0.35,0.55]x_{11} + [0.20,0.40]x_{12} \\ \min Z_4 &= [150,240]x_1 + [140,250]x_2 + [70,130]x_3 + [150,260]x_4 + [95,185]x_5 \\ &\quad + [110,210]x_6 + [150,390]x_7 + [30,70]x_8 + [130,200]x_9 + [90,110]x_{10} \\ &\quad + [120,250]x_{11} + [160,290]x_{12} \end{aligned}$$

Subject to

$$\begin{aligned}
& [64,72]x_1 + [40,50]x_2 + [15,24]x_3 + [38,56]x_4 + [28,42]x_5 + [32,48]x_6 + [55,82]x_7 \\
& \quad + [7,12]x_8 + [35,52]x_9 + [29,44]x_{10} + [11,19]x_{11} + [45,65]x_{12} \leq 380 \\
& [150,240]x_1 + [140,250]x_2 + [70,130]x_3 + [150,260]x_4 + [95,185]x_5 + [110,210]x_6 \\
& \quad + [150,390]x_7 + [30,70]x_8 + [130,200]x_9 + [90,110]x_{10} \\
& \quad + [120,250]x_{11} + [160,290]x_{12} \leq 1700 \\
& t_1x_1 \leq 30, t_2x_2 \leq 30, t_3x_3 \leq 30, t_4x_4 \leq 30, t_5x_5 \leq 30, t_6x_6 \leq 30, t_7x_7 \leq 30, t_8x_8 \\
& \quad \leq 30, t_9x_9 \leq 30, t_{10}x_{10} \leq 30, t_{11}x_{11} \leq 30, t_{12}x_{12} \leq 30 \\
& 0.20x_1 + 0.28x_2 + 0.12x_3 + 0.22x_4 + 0.18x_5 + 0.15x_6 + 0.30x_7 + 0.08x_8 + 0.20x_9 \\
& \quad + 0.18x_{10} + 0.30x_{11} + 0.32x_{12} \geq 1.2 \\
& x_2 + x_3 + x_7 + x_8 \geq 1, \quad x_1 + x_6 + x_{11} + x_{12} \geq 1, \quad x_4 + x_5 + x_9 + x_{10} \geq 2 \\
& \quad x_j \in \{0,1\}, \quad \forall j.
\end{aligned}$$

4.2 Results of the numerical study

Applying the five weighting scenarios according to Table 3, the best solution for each scenario was selected using the ranking method based on the possibility matrix. Table 5 summarizes the findings, including the selected projects, interval values of the raw objectives, total completion time (maximum total completion time among selected projects), and the final total stability score. This problem was implemented in MATLAB (version R2018b) based on the method described in the previous subsection.

Table 5 Summary of findings of the multi-objective model based on weighting scenarios.

Scenario	Selected project indices	Objective values: $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4$	Time	$\sum_j r_j x_j$
S_1	1,2,3,4,5,8,9,10,11	[1.41,3.43], [267,371], [4.55,6.35], [975,1695]	24	1.76
S_2	2,3,4,8,9,11	[0.86,2.10], [146,213], [3.20,4.40], [640,1160]	24	1.20
S_3	1,3,4,5,6,8,9,10,11	[1.31,3.23], [259,369], [4.65,6.45], [945,1655]	24	1.63
S_4	2,5,8,9,10,11	[0.81,2.08], [150,219], [2.95,4.15], [605,1065]	22	1.22
S_5	3,4,5,8,9,10,11	[0.96,2.38], [163,249], [4.20,5.60], [685,1205]	24	1.28

The data in Table 5 show that in the first scenario, the highest resilience values [1.41,3.43] are obtained compared to the other weighting scenarios. Equity is also at a high level [4.55,6.35] (although the third scenario gives higher equity [4.65,6.45]). However, its cost and pollution are also high. This scenario includes 9 projects from all three regions. Its execution time and stability are 24 months and 1.76, respectively. In the economy-oriented scenario, although the lowest cost is achieved, it includes only 6 projects. Resilience and equity here have decreased as expected. Examining the third scenario shows that the results of the equity-oriented scenario are very similar to the first scenario but with a slight increase in equity and a slight decrease in resilience. In the fourth scenario, the lowest pollution and execution time among the weighting scenarios are observed. However, the number of selected projects is only 6, similar to the second scenario. The balanced scenario provides a logical compromise among all objectives. Its values lie in the middle of the other weighting scenarios and it includes 7 projects. It should be noted that no two scenarios have exactly the same set of selected projects.

5 Sensitivity analysis

In any real decision-making model, input parameters are rarely deterministic and fixed. The policymaker may not have complete confidence in the exact values of the budget, carbon cap, minimum reliability, or time intervals. For this reason, sensitivity analysis is an essential tool for evaluating the stability and robustness of non-dominated interval solutions against parameter changes. In this section, the effect of changes in four key parameters, r^{min} (minimum reliability), t^{max} (maximum total completion time), b^{max} (budget ceiling), and e_{cap}^{max} (carbon cap) on the solutions of the weighting scenarios is examined. In each analysis, one parameter is varied over a reasonable range while the other parameters are kept at their base values. The results are reported in Tables 6 to 9.

5.1 Sensitivity analysis with respect to r^{min}

The expected minimum reliability r^{min} is one of the most important technical parameters, directly affecting the number and type of selected projects. As this parameter increases, only those solutions whose total reliability exceeds the new value remain feasible. Table 6 shows the changes in the scenario solutions for different r^{min} values from 1.18 to 1.64.

- At $r^{min} = 1.18$, the feasible region becomes larger. Thus, for the base scenario S_2 , project $P05$ is added and $P09$ is removed, changing the combination to $\{2,3,4,5,8,11\}$. Cost decreases and resilience also drops slightly. In scenario S_4 , projects $P03$ and $P04$ replace $P05$ and $P09$, and pollution decreases slightly. Other weighting scenarios do not change (because they had already selected projects with higher reliability).
- At $r^{min} = 1.21$, only scenario S_2 changes. Projects $P10$ and $P05$ are added instead of projects 3 and 4, resulting in the combination $\{2,5,8,9,10,11\}$. Resilience and cost also increase slightly in this case.
- At $r^{min} = 1.23$, both scenarios S_2 and S_4 change. In S_2 , project $P05$ is replaced by $P04$, and in S_4 , $P04$ also enters. These changes lead to an increase in cost and resilience.
- At $r^{min} = 1.29$, all scenarios except the first one undergoes changes. However, scenarios S_2 and S_4 reach the solution $\{2,3,5,8,9,10,11\}$, and scenarios S_3 and S_5 reach the solution $\{3,4,5,6,8,9,10,11\}$. Cost and pollution in these scenarios increase noticeably, but resilience also jumps.
- At $r^{min} = 1.64$, S_2 and S_4 change again. Project $P07$ enters and project $P03$ is retained. The total completion time reaches 30 months due to the selection of $P07$. In addition, cost and pollution reach their maximum in this analysis.

Table 6 Stability sensitivity analysis based on weighting scenarios.

r^{min}	Scenario	Selected project indices	Objective values: $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4$	Time	$\sum_j r_j x_j$
1.18	S_2	2,3,4,5,8,11	[0.83,2.08], [139,203], [3.20,4.40], [605,1145]	24	1.18
	S_4	2,3,4,8,10,11	[0.83,2.06], [140,205], [2.95,4.15], [600,1070]	24	1.18
	Responses of other weighting scenarios remain unchanged.				
1.21	S_2	2,5,8,9,10,11	0.81,2.08,150,219, 2.95,4.15,605,1065	22	1.22
	Responses of other weighting scenarios remain unchanged.				

1.23	S_2, S_4	2,4,5,8,10,11	[0.83,2.11], [153,223], [3.00,4.20], [625,1125]	24	1.24
Responses of other weighting scenarios remain unchanged.					
1.29	S_2, S_4	2,3,5,8,9,10,11	[0.96,2.43], [165,243], [3.60,5.00], [675,1195]	22	1.34
	S_3, S_5	3,4,5,6,8,9,10,11	[1.06,2.68], [195,297], [4.45,6.05], [795,1415]	24	1.43
Responses of other weighting scenarios remain unchanged.					
1.64	S_2, S_4	2,3,5,7,8,9,10,11	[1.21,2.98], [220,325], [3.90,5.50], [825,1585]	30	1.64
	S_3, S_5	2,3,4,5,6,8,9,10,11	[1.26,3.18], [235,347], [4.60,6.40], [935,1665]	24	1.71
Responses of other weighting scenarios remain unchanged.					

Scenario S_1 is completely stable against increases in $r^{\min}$, because from the beginning it had selected projects with high reliability $\{P01, P03, P04, P05, P08, P09, P10, P11\}$. In contrast, S_2 and S_4 are very sensitive; with a small increase in $r^{\min}$, they absorb projects with higher reliability returns, which increases cost and pollution. Scenarios S_3 and S_5 also have moderate sensitivity.

5.2 Sensitivity analysis with respect to $t^{\max}$

Projects are executed in parallel, so the total reconstruction time equals the maximum total completion time among selected projects. The constraint $t^{\max}$ means that no project with a time longer than this value can be selected. Table 7 shows the results of varying $t^{\max}$ from 21 to 32 months.

- At $t^{\max} = 21$, project $P01$ (with time 22 months) is removed from scenarios S_1 and S_3 . Scenario S_5 also experiences more changes, removing projects 4 and 9 and adding 2 and 6. Ultimately, scenarios S_1, S_3 , and S_5 change their combination to $\{2,3,5,6,8,10,11\}$. Their resilience and equity decrease. Scenario S_4 also changes to $\{2,3,5,6,10,11\}$. The execution time for all scenarios decreases to 20 months because the maximum total completion time in this combination is 20 months. Scenario S_2 remains unchanged because it does not include $P01$.
- At $t^{\max} = 23$, as Table 7 shows, scenarios S_1 and S_3 reach the combination $\{1,2,3,5,6,8,9,10,11\}$, which includes $P01$, and the time remains 22 months. Scenarios S_2 and S_4 also change to the same solution $\{2,5,8,9,10,11\}$. Scenario S_5 undergoes fewer changes compared to these scenarios.
- At $t^{\max} = 32$, project $P12$ (with time 32 months) becomes allowed. Due to its high cost and pollution, this project appears only in scenarios S_2 and S_4 . Other scenarios remain unchanged.

Table 7 Sensitivity analysis of maximum total completion time based on weighting scenarios.

$t^{\max}$	Scenario	Selected project indices	Objective values: $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4$	Time	$\sum_j r_j x_j$
21	S_1, S_3, S_5	2,3,5,6,8,10,11	[0.88,2.31], [162,239], [3.15,4.55], [655,1205]	20	1.29
	S_4	2,3,5,6,10,11	[0.80,2.11], [155,227], [2.55,3.75], [625,1135]	20	1.21
Responses of other weighting scenarios remain unchanged.					

23	S_1, S_3	1,2,3,5,6,8,9,10,11	[1.31,3.28], [261,363], [4.05,5.85], [935,1645]	22	1.69
	S_2, S_4	2,5,8,9,10,11	[0.81,2.08], [150,219], [2.95,4.15], [605,1065]	22	1.22
	S_5	3,5,6,8,9,10,11	[0.86,2.23], [157,241], [3.70,5.10], [645,1155]	22	1.21
Responses of other weighting scenarios remain unchanged.					
32	S_2	3,5,8,9,11,12	[0.81,2.00], [141,214], [3.20,4.40], [605,1125]	32	1.20
	S_4	3,8,9,10,11,12	[0.81,1.98], [142,216], [2.95,4.15], [600,1050]	32	1.20
	Responses of other weighting scenarios remain unchanged.				

A time limit lower than 22 months significantly harms longer-duration projects (such as $P01$ and $P12$). Scenarios S_1 and S_3 , which depend on $P01$, suffer under this condition. On the other hand, increasing the time above 30 months only brings project $P12$ into the combination, and that too only affects scenarios S_2 and S_4 . Therefore, the policymaker can select a maximum total completion time of 24 months to obtain a solution close to the base value in Table 5 and achieve a reasonable balance between time and the other objectives.

5.3 Sensitivity analysis with respect to $b^{\max}$

The current budget with a base value of 380 is one of the most restrictive resources in post-disaster reconstruction. Table 8 shows the changes in solutions when the budget is reduced to 218 and increased to 400.

Table 8 Sensitivity analysis of budget based on weighting scenarios.

$b^{\max}$	Scenario	Selected project indices	Objective values: $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4$	Time	$\sum_j r_j x_j$
218	S_1, S_2, S_3, S_4, S_5	2,3,4, 8,9,11	[0.86,2.10], [146,213], [3.20,4.40], [640,1160]	24	1.20
	S_1	2,3,4,5,8,10,11	[0.98,2.46], [168,247], [3.65,5.05], [695,1255]	24	1.36
248	S_3	3,4,5,6,8,10,11	[0.88,2.26], [160,245], [3.75,5.15], [665,1215]	24	1.23
	S_5	3,5,6,8,9,10,11	[0.86,2.23], [157,241], [3.70,5.10], [645,1155]	22	1.21
	Responses of other weighting scenarios remain unchanged.				
368	S_1	1,2,3,4,5, 8,9,10	[1.36,3.25], [256,352], [4.20,5.80], [855,1445]	24	1.46
	S_3	2,3,4,5,6,8,9,10,11	[1.26,3.18], [235,347], [4.60,6.40], [935,1665]	24	1.71
	Responses of other weighting scenarios remain unchanged.				
370	S_1	1,2,3,4,5, 8,9,10	[1.36,3.25], [256,352], [4.20,5.80], [855,1445]	24	1.46
	Responses of other weighting scenarios remain unchanged.				
400	S_1	1,2,3,4,5,6,8,9,10	[1.46,3.55], [288,400], [4.45,6.25], [965,1655]	24	1.61
	Responses of other weighting scenarios remain unchanged.				

- At $b^{\max} = 218$, all scenarios are forced to remove expensive projects. The same combination $\{2,3,4,8,9,11\}$ is obtained for all five scenarios. This indicates that at a very tight budget, the distinctions between scenarios disappear and all converge to a common solution that has the lowest cost. Consequently, resilience and equity decrease sharply.
- At $b^{\max} = 248$, scenario S_1 is the first to exit the common solution, adding projects $P05$ and $P10$ to obtain the combination $\{2,3,4,5,8,10,11\}$. Its resilience and equity increase accordingly. Scenarios S_3 and S_5 also undergo changes. However, S_2 and S_4 remain in their base state.
- At $b^{\max} = 368$, scenario S_1 reaches the combination $\{1,2,3,4,5,8,9,10\}$, which includes $P01$ with a relatively higher cost. In this case, resilience reaches $[1.36,3.25]$ and equity reaches $[4.20,5.80]$. S_3 also shows the solution $\{2,3,4,5,6,8,9,10,11\}$. Scenario S_1 also changes at $b^{\max} = 370$, reaching the same combination $\{1,2,3,4,5,8,9,10\}$.
- At $b^{\max} = 400$, scenario S_1 removes $P11$ and adds $P06$, reaching the combination $\{1,2,3,4,5,6,8,9,10\}$. In this case, resilience reaches $[1.46,3.55]$, which is the highest value in the entire analysis. Other scenarios remain unchanged relative to their initial responses in Table 5.

Overall, budget is the most critical constraint. At very low budgets, all scenarios converge to a single solution. As the budget increases, the priority of scenario S_1 in absorbing expensive but high-return projects (such as $P01$ and $P02$) becomes evident. Scenario S_2 (economy-oriented) does not tend to increase cost even with a higher budget and remains in its minimal combination. This shows that the policymaker can increase resilience and equity by injecting more budget only if he/she shifts priorities toward those objectives (i.e., a non-economy-oriented scenario).

5.4 Sensitivity analysis with respect to $e_{cap}^{\max}$

The environmental constraint has become one of the main concerns of policymakers in recent years. Table 9 shows the changes for different $e_{cap}^{\max}$ values from 1159 to 1805 relative to the current base value of 1700.

Table 9 Sensitivity analysis of pollution based on weighting scenarios.

$e_{cap}^{\max}$	Scenario	Selected project indices	Objective values: $\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4$	Time	$\sum_j r_j x_j$
1159	S_1, S_3, S_5	3,5,6,8,9,10,11	$[0.86,2.23], [157,241], [3.70,5.10], [645,1155]$	22	1.21
	S_2	2,5,8,9,10,11	$[0.81,2.08], [150,219], [2.95,4.15], [605,1065]$	22	1.22
	Responses of other weighting scenarios remain unchanged.				
1204	S_1	1,2,3,5,8,9,10	$[1.16,2.80], [218,296], [3.45,4.85], [705,1185]$	22	1.24
	S_3, S_5	3,5,6,8,9,10,11	$[0.86,2.23], [157,241], [3.70,5.10], [645,1155]$	22	1.21
	S_4	2,5,8,9,10,11	$[0.81,2.08], [150,219], [2.95,4.15], [605,1065]$	22	1.22
	Responses of other weighting scenarios remain unchanged.				
1654	S_1	1,2,3,4,5,8,9,10	$[1.36,3.25], [256,352], [4.20,5.80], [855,1445]$	24	1.46

	S_3	3,4,5,6,8,9,10,11	[1.06,2.68], [195,297], [4.45,6.05], [795,1415]	24	1.43
Responses of other weighting scenarios remain unchanged.					
1694	S_1	1,2,3,4,5,8,9,10	[1.36,3.25], [256,352], [4.20,5.80], [855,1445]	24	1.46
	S_2	2,3,4,8,9,11	[0.86,2.10], [146,213], [3.20,4.40], [640,1160]	24	1.20
Responses of other weighting scenarios remain unchanged.					
1805	S_3	3,4,5,6,7,8,9,10,11	[1.31,3.23], [250,379], [4.75,6.55], [945,1805]	30	1.73
Responses of other weighting scenarios remain unchanged.					

- At $e_{cap}^{max} = 1159$, scenarios S_1, S_3 , and S_5 , which include highly polluting projects (e.g., $P01$ with [150,240] and $P04$ with [150,260]), are forced to remove them. The new combination for these scenarios becomes {3,5,6,8,9,10,11}. Pollution decreases to [645,1155], but resilience and equity also drop. Scenario S_2 changes to {2,5,8,9,10,11} by removing $P03$ and $P04$. In addition, the execution time for all decreases to 22 months.
- At $e_{cap}^{max} = 1204$, scenario S_1 reabsorbs project $P01$ (because the upper bound of $P01$'s pollution is 240, and the total including other projects does not exceed the new cap). With the change of S_1 's combination to {1,2,3,5,8,9,10}, resilience and equity improve. S_3, S_5 , and S_4 remain unchanged compared to the previous state.
- At $e_{cap}^{max} = 1654$ and $e_{cap}^{max} = 1694$, scenario S_1 reaches the same combination {1,2,3,4,5,8,9,10}. However, only at 1654 does scenario S_3 reach {3,4,5,6,8,9,10,11}. On the other hand, at the cap of 1694, S_2 shows the combination {2,3,4,8,9,11}. S_4 and S_5 return to their base states (according to Table 5) for both values.
- At $e_{cap}^{max} = 1805$, the carbon cap increases enough to allow the highly polluting project $P07$. Scenario S_3 takes this opportunity and adds $P07$ to its combination. In addition, the time increases to 30 months due to the selection of $P07$. Resilience and equity reach [1.31,3.23] and [4.75,6.55], respectively.

In conclusion, the carbon constraint strongly affects the selection of polluting projects (especially $P01, P04, P07$). At low caps, e.g., below 1200, the high-resilience scenarios (S_1, S_3, S_5) suffer and are forced to remove key projects. As the cap increases to about 1650, these scenarios return to their base states. At the cap of 1805, the very polluting project $P07$ is absorbed only by the equity-oriented scenario, indicating that this scenario is willing to accept more pollution to increase equity. Scenario S_4 does not absorb $P07$ even at high caps because it conflicts with its priority.

The sensitivity analysis on the four key parameters shows that:

- Scenario S_1 is relatively stable against changes in r^{min} and t^{max} , but very sensitive to changes in b^{max} and e_{cap}^{max} . An increase in budget or carbon cap is quickly absorbed by this scenario, leading to a significant improvement in resilience.
- The economy-oriented scenario is very stable against increases in budget and carbon cap, but sensitive to an increase in r^{min} , forcing it to absorb more expensive projects.
- The environment-oriented scenario behaves similarly to the economy-oriented scenario, but its sensitivity to the carbon constraint is naturally higher.
- The equity-oriented and balanced scenarios have moderate sensitivity and absorb more projects at intermediate budgets and carbon caps.

- v. Under critical conditions (very low budget or very low carbon cap), all scenarios converge to a single solution, indicating that under severe constraints, policy priorities become practically ineffective.

These findings help the policymaker, given the uncertainty in parameter estimates; for example, the possibility of budget cuts or stricter environmental limits, to select a scenario that is more stable against those changes. For instance, if a budget cut is likely, scenario S_2 or S_4 is more appropriate. If an increase in reliability requirements is likely, scenario S_1 is a better choice.

5.5 Model validation

The sensitivity analyses presented in Sections 5.1 through 5.4 serve as a behavioral validation of the proposed model. A model is considered behaviorally valid if its responses to parameter changes are consistent with logical expectations and real-world intuition. We now summarize the key behavioral patterns observed:

- i. Budget sensitivity: As the budget decreases, the model selects fewer and less costly projects, leading to lower resilience and equity. Conversely, increasing the budget allows the model to absorb higher-cost, higher-return projects, improving resilience and equity. This behavior aligns with the logical expectation that more resources enable better outcomes.
- ii. Reliability sensitivity: As the minimum reliability requirement r^{min} increases, the model is forced to select projects with higher reliability gains. This often comes at the cost of higher pollution and budget consumption. The resilience-oriented scenario remains stable under this change, while the economy-oriented scenario shows higher sensitivity. This pattern confirms that the model correctly differentiates between scenarios based on their inherent priorities.
- iii. Carbon cap sensitivity: Tighter carbon caps force the removal of high-polluting projects, which reduces resilience and equity. This behavior reflects the real-world trade-off between environmental sustainability and infrastructure performance. The environment-oriented scenario is naturally more stable under carbon restrictions.
- iv. Time sensitivity: Reducing the maximum allowable time excludes longer-duration projects, which often have higher resilience or equity impacts. This forces the model to select shorter projects, reducing overall system gains. This is consistent with the practical reality that time constraints limit reconstruction options.

In addition to behavioral validation, we also provide a qualitative comparison between our interval-based approach and the conventional defuzzification approach (where intervals are replaced by their mid-points). The proposed approach preserves full interval information, allowing decision-makers to see the range of possible outcomes (e.g., cost between 267 and 371). Defuzzification would collapse this interval to a single point (e.g., 319), discarding information about uncertainty and potentially leading to overconfident or misleading decisions. Thus, the proposed model provides more robust and informative results.

Collectively, the numerical example, comprehensive sensitivity analysis, and behavioral validation confirm that the proposed model is both practically applicable and theoretically consistent. The model behaves logically under various scenarios and parameter changes, and its interval-based design provides richer information for decision-makers compared to conventional defuzzification approaches.

6 Managerial insights and practical implications

The numerical results and sensitivity analyses presented in the previous sections offer several insights that may be useful for decision-makers in post-disaster reconstruction planning. This section summarizes these findings in a practical context.

6.1 Trade-offs, constraints, and their practical implications

The results from the five weighting scenarios confirm that no single portfolio dominates across all objectives. The resilience-oriented scenario achieves higher resilience and equity at the expense of higher cost and pollution, while the economy-oriented and environment-oriented scenarios prioritize cost and pollution, respectively, at the expense of resilience. This implies that explicit priority setting by policymakers is a prerequisite for using the model.

Budget is identified as the most critical constraint. When budget levels become very low, all scenarios converge to a single solution, indicating that under severe resource scarcity, policy preferences become largely ineffective. The practical implication is that, in highly constrained post-disaster conditions, efforts to secure additional resources should take precedence over detailed discussions on priority weights. The carbon cap similarly affects the selection of polluting projects, and the reliability constraint primarily affects the economy-oriented and environment-oriented scenarios by directing them toward more expensive alternatives.

6.2 Scenario selection and practical recommendations

Given the inherent uncertainty in parameter estimates, scenario selection should be guided by plausible risk factors. Table 10 provides a simple decision guide.

Table 10 Recommended weighting scenario under different risk conditions.

Condition	Recommended scenario	Rationale
Likely budget cuts	Economy-oriented (S_2) or Environment-oriented (S_4)	Lowest cost and pollution; stable under budget reduction
Increase in reliability requirements	Resilience-oriented (S_1)	Already selects high-reliability projects
Stricter carbon caps	Environment-oriented (S_4)	Lowest pollution output
Normal conditions with competing priorities	Balanced (S_5)	Reasonable compromise among objectives

Based on these findings, the following practical recommendations are offered: (a) prioritize resource augmentation under extreme budget or carbon constraints; (b) apply regional quota constraints to avoid concentration of projects in limited areas; (c) conduct sensitivity analysis as a routine step to check solution stability; and (d) involve experts in estimating subjective parameters such as equity and resilience. These suggestions indicate that the model can serve as a structured model for evaluating trade-offs and testing the impact of uncertainties, rather than providing a single prescriptive answer.

7 Conclusion and future suggestions

In this research, a multi-objective optimization model with interval data was presented for selecting a post-disaster water supply network reconstruction project portfolio. The model combines four conflicting objectives; maximizing resilience and distributive equity, minimizing cost and pollution, with realistic constraints such as budget, carbon cap, parallel execution time, minimum reliability, and regional quota. Unlike many previous studies that eliminate uncertainty by early defuzzification, here interval information is fully preserved and solutions are compared using the possibility degree [17]. Normalization of objectives was done using the maximum possible value, and final ranking was performed using the possibility degree matrix and the corresponding score. Numerical results on expert data for five weighting scenarios showed:

1. The resilience-oriented scenario gives the highest resilience and equity with high cost and pollution.
2. The economy-oriented scenario has the lowest cost but low resilience and equity.
3. The environment-oriented scenario produces the lowest pollution.
4. The balanced scenario provides a logical compromise among all objectives.

Sensitivity analysis on the four main parameters gives a clear picture of the behavior of different scenarios. The resilience-oriented scenario is completely stable against increases in reliability requirements, because it had already selected projects with high reliability. However, this same scenario is very fragile against budget cuts or carbon cap reductions and quickly suffers performance loss. In contrast, the economy-oriented scenario behaves oppositely: an increase in budget or carbon cap has little effect on it (indicating that it uses the base budget optimally), but if reliability requirements increase, it is forced to absorb more expensive projects. Interestingly, under critical conditions – e.g., when the budget or carbon cap is very low- all scenarios converge to a single solution and the distinctions between priorities practically disappear. A detailed discussion of the managerial implications, including scenario selection under uncertainty and practical recommendations, is provided in Section 6. The main policy-oriented finding is that under severe resource constraints, scenario preferences become secondary to resource augmentation, which should be the primary strategic focus.

For future research, several directions are suggested. At the modeling level, the synergy or interference effects between projects can be incorporated by adding quadratic terms to the objective functions. Adding other objectives such as temporary job creation or modeling uncertainty with interval fuzzy numbers (triangular or trapezoidal) could also enrich the model. At the solution method level, developments for larger-scale problems, combination with multi-criteria decision-making methods [35], and data envelopment analysis-based models [36] can be considered. Also, comparing the possibility-degree-based ranking method with other interval ranking methods could help select the most appropriate one. At the empirical level, application of the model to other critical infrastructures (power, gas, transportation networks [37]) and designing a user-friendly decision support system are suggested.

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