International Journal of Applied Operational Research
Vol. 3, No. 3, pp. 79-93, Summer 2013

Journal homepage: www.ijorlu.ir

# Comparison of Simulated Annealing and Electromagnetic Algorithms for Solution of Extended Portfolio Model

K. Shahanaghi, M. H. Babaei*, M. R. Hamidi, E. Jahani

**Received:** 20 January 2013 ; **Accepted:** 20 May 2013

**Abstract** This paper presents two meta-heuristic algorithms to solve an extended portfolio selection model. The extended model is based on the Markowitz's Model, aiming to minimize investment risk in a specified level of return. In order to get the Markowitz model close to the real conditions, different constraints were embedded on the model which resulted in a discrete and non-convex solution space. However, due to the NP-hard nature of the problem; two meta-heuristic algorithms were used, namely the simulated annealing and electromagnetic algorithms. Comparative result indicated high efficiency of the extended model and the solution presented by the electromagnetic algorithm.

**Keywords:** Portfolio, Meta-heuristic, Electromagnetic Algorithm, Simulated Annealing Algorithm.

## 1 Introduction

Financial markets are among the most major marketplaces for a country whose conditions exert significant impact on real economic sectors and are seriously affected by others (not essentially in short-term). One of the most important elements of financial markets is the Stock Exchange. This element is considered as a formal and organized marketplace for the exchange of shares of company stock under special terms and conditions.

Customers, in general, have three options to analyze shares of this type; 1) fundamental analysis; 2) technical analysis (charting); and 3) portfolio analysis [1]. Unlike the others, the latter is the core to assess risk and return, based on two hypotheses: first, markets are efficient, and second, there are available data for markets and individual stocks [2]. In the current study, this approach is used for analyzing stocks and how to invest. While the analysis of securities investment is discussed in two general frameworks as follows: (1) Analysis of stock selection in an individual manner, ranging from stocks of manufacturing and service firms to shares of investment companies investing on the former and (2) Design of a systematic portfolio.

The first framework applies the fundamental and technical analysis approaches to analyze and select stock. However, the second framework is concerned with the modern portfolio

* **Corresponding Author.** (✉)
**E-mail:** mh_babaei@ind.iust.ac.ir (M. H. Babaei)

**K. Shahanaghi**
Assistant professor, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

**M. H. Babaei, M. R. Hamidi**
Ph.D Student, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

**E. Jahani**
Bs.C Student, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

theory (MPT), i.e. the efficient market hypothesis (EMH) [3]. The current study focuses on portfolio analysis methodology.

"Portfolio" simply means a "basket of investment" generally, and a "basket of stocks" in particular. It consists of a combination of assets holding by an investor, whether an individual or entity. Technically, a portfolio may comprise a complete set of real and financial assets for an investor. Note that this paper concerns just financial assets [2].

Financial modeling has been developed by integrating financial approach and mathematical programming, in response to the need to optimize financial and investment decision-making processes. For portfolio design problem, the main question is that among securities or stocks with given yield and return on investment, which we have to choose in order to ensure an appropriate return, as well as a minimized investment risk.

Regardless of scientific viewpoint to the mean variance model (the Markowitz-type model), it is often a very simple model to provide a proper complexity of stock selection problems. The primary model of a portfolio, for example, evaluates only risk and stock return; while an actual portfolio is affected by many variables in the real world. By undertaking a literature review and addressing stock market behaviors, the current paper seeks to present a more complete model in order to simulate the real world complexity properly. In addition, using the electromagnetic algorithm provides better solutions for the model in a short time interval, compared to other algorithms.

## 2 Assumptions

First, development of a portfolio optimization model is founded on the basic assumptions of the Markowitz model. Investors are sensitive in returns, while uninterested to risk; they also show a rational behavior and decide to maximize their expected utility. Investors' utility, therefore, is a function of expected values of risk and return. Furthermore, stock markets are assumed to act as the efficient market hypothesis where investors determine stock prices according to assets' expected future cash flows and their risks [2]. On the other hand, within the efficient market, available data has an immediate effect on stock prices. The efficient market concept is conceptually rooted in the assumption that investors will consider all relevant data to define stock prices, when deciding to trade. Therefore, present stock prices include all known data ranging from the past (e.g. last season's/year's yield) to the present [3].

## 3 Literature review

Given the importance of optimal portfolio selection problem and its application in today's world, numerous articles have been published about this field. In his paper, Chen represented return rates and risk by triangular fuzzy numbers. Also, based on four major indices, including rates of return, risk, turnover rate and Treynor index, he divided stocks into four groups namely efficiency, stable-value, aggressive, and good efficiency funds. His fuzzy model tried to minimize investment risks and simultaneously maximize return rates, by defining investment ratio in each group. Among these four groups, good efficiency funds dominated the others so that only this group was entered into the model. Moreover, Chen found that fuzzy operation obtained better results for the return rates and risks, because of its uncertainty [4].

Lin and Liu presented six models based on the Markowitz model and used a multi-criteria decision making (MCDM) approach by concerning the nature of problem. One model was developed according to fuzzy numbers and the MCDM. As they indicated, a decision-maker can describe his own priorities based on weighted values of return and risk; as well provide results as close as possible to the real world. It should be noted that Lin and Liu applied a simulated annealing algorithm to solve the model [5].

Chang and the colleagues showed that how restricted numbers of stocks selected by an investor could make an efficient frontier discontinuous. In addition to constraints of the Markowitz model, they added numbers of portfolio stocks to their research. By using three meta-heuristic algorithms, i.e. TABU research, simulated annealing, and gradual freezing, research findings indicated that the simulated annealing algorithm can provide a better result at scales with more than 100 stocks in sizes [6]. In sum, research contents conducted on portfolio problems based on Markowitz's modern theory can be divided into two general stages:

1. To develop new model, and
2. To solve model

### 3.1 Developing a new model

Markowitz's modern portfolio theory has provided a novel model of portfolio selection problem for investors in terms of forming portfolio with the highest expected return (yield) at a given level of risk or with the minimal risk at a given level of return [7, 8, 9]. Great attempts have been devoted to solve and extend the Markowitz model, resulting in more practical model with regards to real market constraints.

Markowitz (1956) developed the critical linear technique to solve his quadratic model [10]. Wolfe tried to solve the model by the simplex approach [11]. Then, Markowitz presented more detailed studies by the semi-variance method [12].

Speranza (1993) introduced a more general model with weighted risk function for the first time [1], also presented an integrated planning model by concerning actual characteristics of portfolio selection like the minimum amount of transactions and the maximum number of portfolio stocks. Further research considered different constraints or restrictions to the Markowitz model. Yoshimoto investigated a multi-period portfolio selection problem with transaction costs based on the Markowitz model [13]. Kono (2001) proposed a new algorithm to solve portfolio optimization model with regards to transaction costs and minimized trading volumes [14].

### 3.2 Model solution

For solving models, Arnone presented the simulated annealing algorithm for the non-constrained optimization problem [15]. However, Shoaf applied this algorithm on the Markowitz model without any further constraint for the first time [16]. Furthermore, Rowland used the TABU search algorithm to solve the model [17].

In the early 1900's, several studies found out more functions for meta-heuristic algorithms to solve portfolio selection problems. In order to present better efficiency of such algorithms, Khia and the colleagues (2000) [18], Arito and the colleagues (2003) [19], Fidsand and the colleagues (2004) [20] and Lai and the colleagues (2006) [51]; also Chang and the colleagues [22] investigated a wide range of meta-heuristics, including TABU search, gradual freezing and simulated annealing algorithms to solve the portfolio selection model without trading volume and business turnover constraints. Chang and the colleagues achieved to the best approximation of the simulated annealing algorithm at a non-constrained frontier; however, they could not develop a meta-heuristic instance better than others. Scarf first improve Chang's paper [23], and then proposed a TABU search algorithm as a solution for Markowitz's model [24]. Lin and the colleagues (2001) evaluated a multi-objective simulated annealing algorithm for portfolio selection problem [25]. By applying the gradual freezing to solve the model with investment return constraint [26], kerma and the colleagues introduced a new portfolio selection algorithm based on Beta portfolio by considering investment sectors. He chose the simulated annealing algorithm to solve his model [27]. Stein extracted several meta-heuristics algorithms for the Markowitz model [28]. Trends to the Markowitz model and its solution by meta-heuristic algorithms continued until 2007. Lin and Liu (2007) assessed the model through minimizing trading volume and presented three additional models. They also used the genetic algorithm to solve their proposed models [29].

## 4 The Proposed Model

In early 1950's, Harry Markowitz founded a basic portfolio model portfolio on which the modern portfolio theory is based. He was the first who developed the concept of diversification for the investment basket generally, and for portfolio in particular, in a formal manner. He quantitatively showed that why and how investment risk could be reduced by the diversification for investors. In order to extend his model, Markowitz presented some basic assumptions; investors 1) are interested to return and insensitive to risk, 2) to make decisions have rational behavior, and 3) decide based on their maximized utility. Hence, an investor's utility is a function of risk for expected return, two major parameters in investment decisions. Markowitz's model is based on relationships between these two necessary variables, i.e. risk and return.

In his initial model, Markowitz assumed that investors' main objective was to maximize return rates for a given amount of risk or to minimize risks for a given level of return. Typically, a decision maker considers a fixed rate of return and, then, minimizes portfolio risk under return constraint. So the Markowitz model can be modeled as a quadratic programming problem as the following [7]:

Where,
$n$ = number of stocks
$r_p$= expected rates of return on investment
$r$= required rate of return
$r_i$= expected rates of return on stock i
$\delta_{ij}$= covariance between the returns on two stocks $i$ and $j$
$\delta_p^2$= portfolio return variance
$w_i$= investment-capital stock ratio.

According to the Markowitz model, the portfolio risk depends on three different factors: variance of individual stock, covariance between stocks, and weights (percentage of invested funds) given to a single stock. Hence, the portfolio risk includes not only a single stock risk (variance), but also the covariance between two stocks. "Covariance" may have the same importance as integration of individual stock risks. When a stock is added to a portfolio, therefore, the mean covariance between that stock and the others available in the portfolio will be of more importance than its risk.

As stated before, the expected value of the stock desired is one of the portfolio selection criteria. In order to evaluate this criterion, the stock price to return ratio, presented by $P/E$ , is considered. Generally, to select a more suitable portfolio, stocks should be chosen such that total $P/E$ ratio will be low; however, according to the above-mentioned descriptions, a low $P/E$ -based investment strategy will not show a high degree of confidence. Furthermore, the $P/E$ ratio varies in time, so a low ratio selection strategy should be applied carefully. The reason is that such ratio is affected by the business cycles and interest rates. However, the $P/E$ ratios for various industries depend on the type of industry. For industries with fast growth and high technology, for example, a high $P/E$ is common. On the contrary, financial institutions rarely offer such ratio. In other words, companies active in young and new industries which have fast growth provide a higher $P/E$ ratio, because their profit growth are rarely maintained; on the other hand, companies active in matured industries which have lower growth provide less $P/E$.(Rau and Vermaelen 1998)

Given the above description and shortcomings of the low $P/E$ -based investment strategy, the present study tried first to classify the stocks based on a low $P/E$ ratio, then applied some constraints in order to maximize profitability. Among such constraints for the initial model is that portfolio stocks are selected from different industries. Numbers of industry must be greater than the minimum level predetermined for the model. This reduces impact of different parameters affecting on low $P/E$ stock strategy such as bank interest, industry types and so on; resulting in a portfolio more safe and close to the real world. The model is given as below formula:

As seen, equation (4) is the objective function minimizing selection risk. Equation (5) ensures that the total investment to fund ratio is equal to 1. Equation (6) guarantees minimum profit return. Equation (7) ensures that K number of stocks is exactly places in the portfolio offered. Considering equations (8); if stock $i$ is placed in the portfolio, then the binary variable $z_i$ will equal to 1, and otherwise to 0. Considering equation (9); If no part of the $j$ is selected, then $z_i$ will be 0, also $y_{ji}$ is 0. However, even if a portion of the $j$ is selected, $y_{ji}$ must be greater than 0. (Indeed, it must be greater than $\frac{w_i}{M}$), it will be equal to 1 because it is a binary variable.

In other words, when some stocks are selected from different groups, then total weight of each batch invested in those groups should be relevant to its investment sector. A group with higher investment priority, if stocks selected from, should have more stock share in the final portfolio. Therefore, the important note is that the temporal constraint is failed when no stock from a group is selected. For example, when group# 1 has more profitability than group#2 and there are available stocks from both, then the first group achieves a total weight greater than the other. However, if no stock from this group (#1) is placed in the portfolio, then the constraint would be failed. Equations (10) and (11) clearly show this. Equation (12) meets investment constraint for minimum investment group G.

## 5 An electromagnetic algorithm as model solution

This section deals with the solution of the proposed model using the new meta-heuristic electromagnetic algorithm. First, the algorithm is introduced with the implementation steps. Next, the model solution and data analysis will be discussed.

### 5.1 Electromagnetic algorithms introduction

Electromagnetic algorithm is used to solve optimization problems. The algorithm utilizes the attraction - repulsion characteristics of charged particles. In this algorithm, each answer is considered as a charged particle. Particles with better objective functions have more charges, so they can attract other particles. Consequently, particles with low optimization repulse other particles. The main idea of the algorithm is that better position may be found around good points. So, weak points are moved toward optimal points [30].

Heuristic electromagnetic algorithm consists of four steps; primary population production, local search, force vector calculations, and displacement toward the force vector applied and using a local search in neighborhood to achieve local optimization. Following is a brief review of the steps [31].

#### 5.1.1 Procedure to Produce Primary Population

The procedure for primary population production is used to generate point $m$ in a possible space where each point has $n$ dimensions and each dimension must be located within high and low limits of that point. When a point obtained in the space, its objective function is calculated. By determining all points and retaining the point with best objective function in $x^{best}$, the function will come to an end.

#### 5.1.2 Local search

Local search procedure includes local data collection around the point $x^i$. Parameters $LSTTER$ and δ applied here represent frequency (the number of iteration) and diffusion coefficient in the neighborhood, respectively.

#### 5.1.3 Total force vector calculation

Electromagnetic Superposition principle states that the force applied on a point from other points has an inverse relationship with distance between points, while having a direct relationship with their charges. For iteration, points' charges are measured through their objective functions. In the current heuristic approach, points' charges vary in each step. Charge level of point $i$ determines his attraction or repulsion power, given as below:

Thus, points with better objective functions will also have more charges. $n$ 's coefficient as problem dimension is applied on the fraction; since for greater dimensions which need more points, the fraction may be so small, resulting in difficult calculation of charge. So this application prevents such a difficulty. Unlike real charges, here the charges have no sign.

Rather, charge direction between two given points is determined based on their objective functions. Therefore, total force vector $F^i$ applied on point $i$ is calculated as below:

As it is obvious, between two points, that point with better objective function attracts other point. In contrast, a point with worst objective function repulses the other. Since point $x^{best}$ has the lowest objective function, this points acts as an absolute attraction point and attracts other points in the population.

### 5.1.4 Displacement toward total force vector

When total force vector $F^{ij}$ calculated, point $i$ moved by random step lengths, directed to the force vector. Here, random step length, λ , is supposed to have a uniform distribution between 0 and 1. Different distributions can be considered for step lengths; however, to simplify calculations and programming, a uniform distribution function is used. Step lengths are selected randomly in order to make all space movement possible. For equation (15), vector $RNG$ causes movements done in a possible space and dimensions move in their high and low limits. In addition, forces applied on the points are normalized; this provides points' stability in the possible space:

Note that there is no displacement for the best point $x^{best}$, without any change it transfers to next step. In order to reduce program time, therefore, calculating the force applied on $x^{best}$ can be ignored.

Here, electromagnetic algorithm ends. In next sections, some modifications are conducted on the algorithm in order to solve the portfolio selection problem.

### 5.2 Algorithm implementation

Based on the provided model and the algorithm steps described above, it is the time to solve the problem. First, samples are considered as the vector$1 * n$, where $n$ is total numbers of shares. Equal to stocks selected, positive values (between 0 and 1) are allocated to the vector elements that represent investment percent for the stock desired. Furthermore, the sum of these positive values will be equal to 1; as the following example:

[0.00, 0.00, 0.45, 0.00, 0.20, 0.10, 0.00, 0.20, 0.00, 0.05]

where total number of stocks is 10, and total stocks of portfolio are 5. In this algorithm, number 10 is the constraint for stocks to be invested. Initial answers are also randomly chosen. When an initial answer generated, next step begins; i.e. the local search. This step improves available answers, without replacing the portfolio stocks with alternatives. The operation is applied only on the weights. It must be conducted in such a way that the possible space is maintained, while changing answers to relatively improve them. On the other hand, due to grouping constraint, if stocks are selected from a group, then the sum of stock weights in that group should be greater than in lower groups. In order to fulfill this constraint, therefore, initial answers are multiplied to dimensions of portfolio stocks to obtain the best answer in a square matrix. The result would be an answer set with possible goodness more than previous ones. Following criteria is considered for the matrix with regards to possibility of initial answers:

- The sum of coefficients must be constant (equal to 1); 2- Grouping constraint is not violated.

For this, the square matrix must be under the following conditions:

- Sum value of counts in each row equals 1. 2- count weights in each row must be descending from left to Right.
- In order to update the matrix used for local search, two or more rows can be changed or replaced by new row.

### 5.3 Computational result

Before evaluating answers obtained from the electromagnetic algorithm, the algorithm application to solve the portfolio problem is firstly compared between the Markowitz and the present models. Note that values of parameters and some controlled variables must be determined. The problem parameters are number of initial answers, local search, and number of iterations; while the control variable only is number of shares selected by an investor. To achieve proper values, a nominal example is used to calculate optimal values based on which, then, a real problem is solved.

#### 5.3.1 Initial population

Obviously, more numbers the initial samples have; more accuracy the answers have. However, increased initial population results to an increase in algorithm implementation time. So, it is necessary to make a balance between implementation time and implementation accuracy (number of initial answers). To do this, efficiency of different numbers for initial population is evaluated with a given stock number in each iteration. The following diagram shows the effect of initial population on the objective function in a problem consisting of 15 stocks and 100 iterations, indicating that the optimum number for the initial answer equals to 40 stocks.

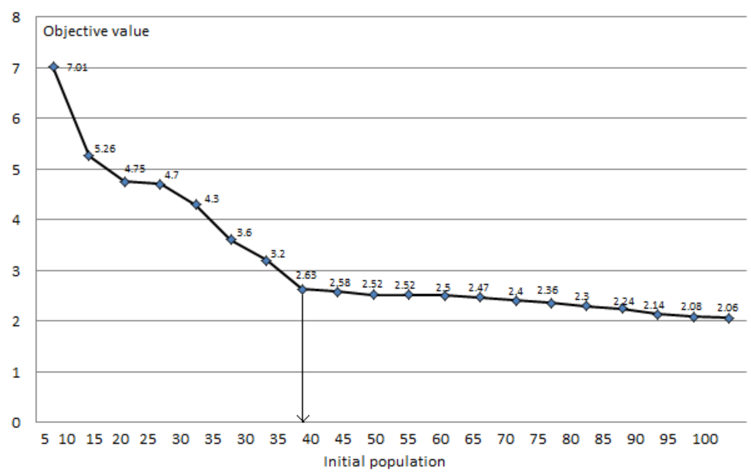


**Fig. 1** Optimum Conditions for Initial Population

Effect of the availability or lack of local search: To evaluate impact of local search in the algorithm, three modes of 10, 30 and 50 are provided for the initial population. Ultimately, local search efficiency on algorithm implementation is proved.

**Table 1**. Comparative Results for Algorithms with and without Local Search; Objective Function at Scale of 1*1000

Considering to relatively optimal calculation for each parameters, optimal values for the parameters can be achieved through a comprehensive and integrated testing method. For this purpose, the below table is formulated:

**Table 2**. Optimal Solution for Optimal Parameters of Proposed Algorithm

According to computations result, the best condition for algorithm parameters includes 40 stocks for initial population, 50 numbers for iteration, with availability of local search constraint. Note, portfolio stock number is considered by 5.

Effect of the number of stocks selected by an investor: as described for the Markowitz model, the main reason to develop a portfolio is to minimize investment risk by adding those stocks with low or inverse coefficient of positive correlation related to available stocks. It is expected that increased portfolio stocks leads to decreased portfolio risk, or at least to non high increase. The following diagram shows the results:

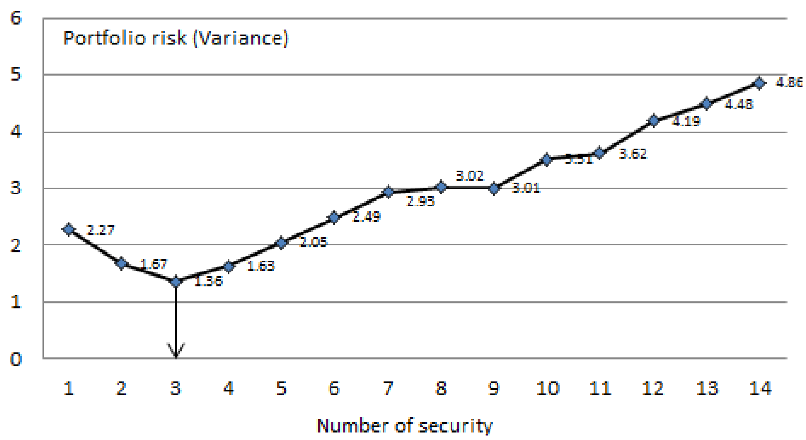


**Fig. 2** Variations in Portfolio Risk resulted from Changes of Portfolio stock Number at Scale of 1*1000

As seen from Diagram 2, the optimal condition for the portfolio stocks is 5.

### 5.3.2 The computational result

The number of shares discussed in the paper is 50. The Stock Exchange was considered as an elite reference for selecting stocks. Sample includes stocks of 50 active or top companies introduced by the Stock Exchange. Relevant data was gathered from the official website of the Tehran Stock Exchange (http://www.tsetmc.ir).

A software was designed to be used for collecting and analyzing data, as well as for computing input data. This software can receive data directly from the website as xml files (http://ww2.tsetmc.com/WebService/TsePublic.asmx), and then automatically calculates algorithmic input data including monthly and total (annual) return rates for each stock, and covariance between stocks. The software was programmed in .NET environment in C#. As stated earlier, the proposed model was investigated in two steps. First, the model was compared with the Markowitz model by using the electromagnetic algorithm. Computational result is represented at the following table:

**Table 3** Best Objective Function obtained from Proposed and Markowitz Models (Scale of 1*1000)

| | | **Based on year 1387** | **Based on the first half of year 1388** |
|---|---|---|---|
| | **Risk (objective function)** | **Rate of Return** | **Rate of Return** |
| **Proposed Model** | 0.53 | 0.43 | 0.29 |
| **Markowitz Model** | 0.58 | 0.26 | 0.21 |

As seen from Table 3, Implementing the Markowitz and proposed models on data for period (March, 21th -Sep. 21th, 2009) shows a better return rate for the proposed model.

**Table 4** List of Companies Used for Portfolio Stocks and Proposed Model

| **Proposed Model** | | | **Markowitz Model** | |
|---|---|---|---|---|
| Industry Group | Security Weight | Company Name | Security Weight | Company Name |
| Automotive and Parts Manufacturing | 0.22 | Zamyad | 0.33 | Iran Khodro Dizel |
| Automotive and Parts Manufacturing | 0.27 | Iran Khodro Dizel | 0.87 | Sanaye Joushkabe Yazd |
| Machinery and Equipment | 0.1 | Tolide Tajhizate Sangine Hepco | 0.06 | Kimi Daroo |
| The Banks and Credit Institutions | 0.12 | Sina Bank | 0.49 | Foolade Amir Kabire Kashan |
| Pharmaceutical Products | 0.27 | Alborz Daroo | 0.02 | Mese Shahid Bahonar |

**Table 5** Number of Stocks and Investment Rates of Groups

| | **Group Number** | **Investment Portion in Industries** | **Security Number After Ranking Based on P/E** |
|---|---|---|---|
| **Proposed Model** | 1 | 0.606 | 4 |
| | 2 | 0 | 5 |
| | 3 | 0 | 7 |
| | 4 | 0.393 | 36 |
| | 5 | 0 | 37 |
| **Markowitz's Model** | 1 | 0.33 | 5 |
| | 2 | 0.087 | 12 |
| | 3 | 0.059 | 23 |
| | 4 | 0 | 44 |
| | 5 | 0.51 | 46 |

### 5.4 Comparison of Proposed Electromagnetic and Simulated Annealing Algorithms

The efficiency of the proposed model was obtained against the Markowitz model. Now, another meta-heuristic approach, namely the simulated annealing algorithm, is used to solve the problem.

Simulated annealing algorithm is a meta-heuristic local search method for problem optimization. This method is mostly applied for discrete optimization problems, rather than continuous ones. An important feature is that this method provides useful means to ignore local optimal points, and find a general optimal one by accepting worse solutions with a given level of probability. Structure of this algorithm was first developed by Kirk and the colleagues in 1983[32]. Freezing phenomena was the main idea for this method, aimed to reduce material temperature to lowest energy level. In freezing process, material with an initial temperature is slowly cooled and the cooling process stops at a final temperature. So, material energy level is gradually decreased and ultimately stopped. Patrick used such energy level as a value for objective functions in different problems.

The purpose of this section is to compare the electromagnetic and simulated annealing algorithms in terms of problem solution in order to prove first's priority over the latter. Obviously, due to different operational steps in each algorithms and different procedures to produce primary populations, it seems irrational to compare the algorithms in terms of iterations. Consequently, we tried to compare problem answers during a same period time of implementation.

The general procedure in simulated annealing algorithms is shown as the following figure.

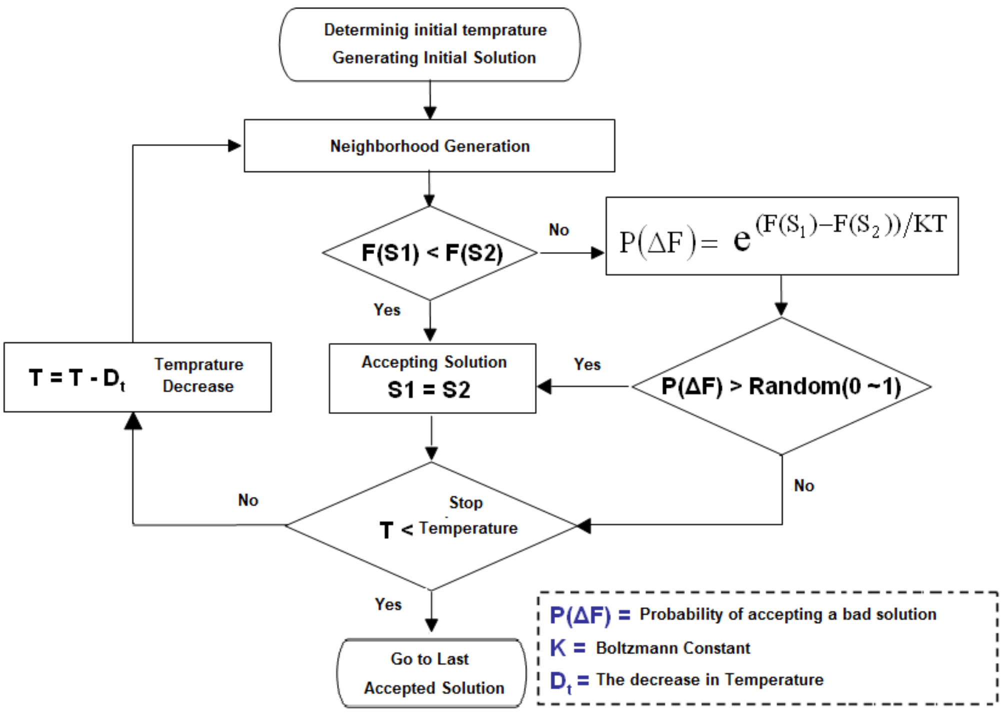


**Fig. 3** Simulated annealing algorithm

At below, the problem solution is discussed about the simulated annealing algorithm.

As mentioned before, the stock selection constraint is 5. For algorithm SA, the initial answer is selected among 5 stocks with much greater yield rate in the current year. Note, a proper initial answer will have a significant effect on the convergence rate. Similarly, the neighborhood process is also important. In order to create a new neighborhood, a criterion is first determined for each stock; this is the ratio of return rate to the variance of each stock. Naturally, the higher the rate is, the higher the expected dividend is in the long term. According to the description mentioned, the ratio is calculated for each portfolio stock based on which, then, the stocks are arranged in descending order from 1 to 5. A selection probability is allocated to stocks according to their ranks. A stock in third rank, for example, is selected by a probability of 0.2 (3/15 = 0.2) as withdrawing sock. It is clear that the possibility of withdrawing each share is proportional to its rank. Next, stocks entering to portfolio are randomly selected among other available stocks.

The initial value of system temperature exerts a direct impact on acceptance or failure of answers. Since for high system temperature, the system energy is also high; this is a desired condition to obtain a best temperature reduction method for in order to achieve a steady-state system. When the initial temperature is low, worst answers have less probability to accept and the system may remain in the local optimum. White (1984) presented the idea of initial temperature equality to the standard deviation of system costs from the average costs [33]. As he stated, initial temperature put in equal to the standard deviations of objective values per times of program implementation in a non-steady states where *OBJ(j)* is the value of objective function in terms of the answer *j*.

Standard criteria for temperature reduction and cooling the system are given as the following function;

With regards to problem aspects, it should be mentioned that the coefficient $\alpha$ is 0.9. In addition, stopping criteria is necessary to define. As stated before, algorithms are to be implemented in the same period time in order to make comparison possible, because of their differences and irrational comparison. Therefore, stopping criteria is not defined for the algorithm, and it continues indefinitely unless a pre-determined period ends.

**Table 6** Comparison of Proposed and Simulated Annealing Algorithms

| | | **Best Objective Function** | | **Solutions Average** | | **Solutions Variance** | | **Average Number of Iterations** | |
|---|---|---|---|---|---|---|---|---|---|
| **Initial Population** | **Time (Seconds)** | **SA** | **EM** | **SA** | **EM** | **SA** | **EM** | **SA** | **EM** |
| **30** | 10 | 1.25 | 1.12 | 2.24 | 1.76 | 0.69 | 0.3 | 723 | 17 |
| | 20 | 1.12 | 1.02 | 1.87 | 1.69 | 0.34 | 0.28 | 1482 | 36 |
| | 30 | 1.01 | 0.95 | 1.71 | 1.62 | 0.14 | 0.23 | 2430 | 48 |
| **40** | 10 | 1.24 | 1.1 | 2.18 | 2.11 | 0.45 | 0.28 | 647 | 11 |
| | 20 | 1.01 | 1 | 1.86 | 1.84 | 0.3 | 0.24 | 1363 | 20 |
| | 30 | 0.98 | 0.83 | 1.69 | 1.75 | 0.22 | 0.21 | 2284 | 27 |
| **50** | 10 | 0.18 | 1.08 | 2.15 | 2.12 | 0.31 | 0.26 | 532 | 8 |
| | 20 | 1 | 1.01 | 1.83 | 1.83 | 0.26 | 0.2 | 1212 | 14 |
| | 30 | 0.93 | 0.84 | 1.65 | 1.77 | 0.22 | 0.21 | 1994 | 19 |

As found from Table 6, the proposed model provides a better performance, compared to the simulated annealing algorithm. Low variance for answers obtained from the proposed model indicates the same answers in each iteration. In other words, such answers show low deviations from the average answers through iterations, unlike the simulated annealing algorithms.

For the proposed algorithm, the main strong feature is to produce an optimum population in less iteration times. Table 7 compares the proposed and simulated annealing algorithms in terms of variations in the average objective function for present and new populations. Result shows that the proposed algorithm has best capability to achieve an optimum population through small numbers of iterations. Hence, it can be used as a combining algorithm to obtain a relatively optimal population through less iteration numbers. Note that the software used here is designed so that each algorithm can apply the population offered by other algorithm with any iteration desired.

**Table 7** Variations in Population Objective Function resulted from Proposed and Simulated Annealing Algorithms

| | | | **Average of New Population Objective Function** | |
|---|---|---|---|---|
| **Initial Population** | **Average Current Population** | **Time (Seconds)** | **SA** | **EM** |
| 30 | 53.89 | 10 | 25.34 | 3.93 |
| | | 20 | 19.87 | 2.7 |
| 40 | 56.21 | 10 | 28.75 | 2.45 |
| | | 20 | 21.03 | 2.19 |
| 50 | 55.98 | 10 | 24.69 | 3.5 |
| | | 20 | 19.34 | 2.86 |

## 6 Conclusions

In the present study, a new model was extended based on the Markowitz model. In general, for portfolio problems, stocks were selected so that they have less risk, while obtaining a given profitability. Extension of the basic model and variations in solution space due to additional constraints led to a discrete and non-convex space. Therefore, the meta-heuristic electromagnetic algorithm was used to solve the problem. The solution process was conducted as the following: stocks were first arranged ascending according to the price to income ratio variable. Then, stocks were divided into five groups, as the highest group had the lowest price to income ratio, and vice versa. The applied constrain indicated that when one or more stock were selected from high groups, the sum of stocks in that group should be greater than in lower groups. Such modeling made it possible to select stocks from different industries, ensuring to pursuit the stock selection strategy with lowest price to income ratio. Finally, the proposed and Markowitz models were solved in order to evaluate efficiency of the present model. When the efficiency determined, two methods were discussed, including electromagnetic and simulated annealing algorithms. Computational result indicated a better efficiency for the electromagnetic algorithm to solve the problem.

## References


1. Speranza, M. G., (1993). Linear programming models for portfolio optimization. Finance, 14, 107–123.
2. Jones, C. H. P., (2009). Iinvestment: analysis and management. Wiley 11 edition, 234-239.
3. Chen, L. H., Huang, L., (2009). Portfolio optimization of equity mutual funds with fuzzy return rates and risks, 36, 3720–3727.
4. Lin, C. C., Liu, T. Y., (2008). Genetic algorithms for portfolio selection problems with minimum transaction lots. European Journal of Operational Research, 185, 393–404.
5. Chang, T. J., Meade, N., Beasley, J. E., & Sharaiha, Y. M., (2000). Heuristics for cardinality constrained portfolio optimization. Computers and Operations Research, 27, 1271–1302.
6. Markowitz, H., (1952). Portfolio selection. Journal of Finance, 7, 77–91.
7. Markowitz, H., (1959). Portfolio selection: Efficient diversification of investments. New York: Wiley.
8. Oh, K. J., Kim, T. Y., Min, S. H., Lee, H. Y., (2006). Portfolio algorithm based on portfolio beta using genetic algorithm. Expert Systems with Applications, 30, 527–534.
9. Markowitz, H., (1956). The optimization of a quadratic function subject to linear constraints. Naval Research Logistics Quarterly, 3, 111–133.
10. Wolfe, P., (1959). The simplex method for quadratic programming. Econometrica, 27,382–398.
11. Markowitz, H., Todd, P., Xu, G., Yamane, Y., (1993). Computation of mean-semivariance efficient sets by the critical line algorithm. Annals of Operations Research, 45, 307–317.
12. Yoshimoto, A., (1996). The mean–variance approach to portfolio optimization subject to transaction costs. Journal of the Operations Research Society of Japan, 39(1), 99–117.
13. Konno, H., (1990), Piecewise linear risk-function and portfolio optimization. Journal of the Operations Research Society of Japan, 33, 139–156.
14. Arnone, S., Loraschi, A., Tettamanzi, A., (1993). A genetic approach to portfolio selection. Neural Network World, 6, 597–604.
15. Shoaf, J., Foster, J. A., (1996). The efficient set GA for stock portfolios. In Proceedings of the Decision Science Institute, Orlando, 571–573.
16. Rolland, E., (1997). A tabu search method for constrained real-number search: Applications to portfolio selection. Technical Report, Department of Accounting & Management Information Systems. Ohio State University, Columbus.
17. Xia, Y., Liu, B., Wang, S., Lai, K. K., (2000). A model for portfolio selection with order of expected returns. Computers and Operations Research, 27, 409–422.
18. Orito, Y., Yamamoto, H., Yamazaki, G., (2003). Index fund selections with genetic algorithms and heuristic classifications. Computers and Industrial Engineering, 45, 97–109.
19. Fieldsend, J., Matatko, J., Peng, (2004). Cardinality constrained portfolio optimization. In Proceedings of the fifth international conference on intelligent data engineering and automated learning (IDEAL'04), Exeter, 25–27 August.
20. Lai, K. K., Yu, L., Wang, S., Zhou, C., (2006). A double-stage genetic optimization algorithm for portfolio selection. College of Business Administration, Hunan University.
21. Chang, T. J., Meade, N., Beasley, J. E., Sharaiha, Y. M. (2000). Heuristics for cardinality constrained portfolio optimization. Computers and Operations Research, 27, 1271–1302.
22. Schaerf, A., (2002a). Local search techniques for constrained portfolio selection problems. Computational Economics, 20, 177–190.
23. Schaerf, A., (2002b). Local Search Techniques for Constrained Portfolio Selection Problems. Dipartimento di Ingegneria Elettrica, Gestionale e Meccanica, Università di Udine. Computational Economics, 20, 177–190.
24. Lin, D., Wang, S., Yan, H., (2001). A multiobjective genetic algorithm for portfolio selection problem. In Proceedings of ICOTA, Hong Kong, 15–17 December 2001.
25. Crama, Y., Schyns, (2003). Simulated annealing for complex portfolio selection problems. European Journal of Operational Research, 150, 546–571.
26. Oh, K. J., Kim, T. Y., Min, S., (2005). Using genetic algorithm to support portfolio optimization for index fund management. Expert Systems with Applications, 28, 371–379.

27. Stein, M., Branke, J., Schmeck, H., (2005). Portfolio selection: How to integrate complex constraints. Institute for Applied Informatics and Formal Description Methods. University of Karlsruhe (TH), June 1.
28. Lin, C. C., Liu, T. Y., (2007). Genetic algorithms for portfolio selection problems with minimum transaction lots. European Journal of Operational Research. O.R. Applications.
29. Rau, P. Raghavendra and Vermaelen, Theo (1998), 'Glamour, value and the post-acquisition performance of acquiring ', Journal of Financial Economics, 49 223-53.
30. Ilker, S., Birbil, F., Fang, S.Ch., (2003). An Electromagnetism-like Mechanism for Global Optimization, Journal of Global Optimization 25,263-282.